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Thermophysical Properties of Some Key Solids: An Update

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In 1985, the CODATA Bulletin published a Report of its Task Group on Thermophysical Properties of Solids which analyzed available data on, and gave recommended values for, the heat capacity of Cu, Fe, W, and Al_2O_3, the thermal expansion of Cu, Si, W, and Al_2O_3, the electrical resistivity of Cu, Fe, Pt, and W, the thermal conductivity of Al, Cu, Fe, and W, and the absolute thermopower of Pb, Cu, Pt, and W. The analysts for the different properties were R. B. Castanet, S. J. Collocott, P. D. Desai, C. Y. Ho, J. G. Hust, R. B. Roberts, C. A. Swenson, and G. K. White. The present paper is an updated version of the earlier report and includes more recent data which change some of the recommended values, notably the heat capacity of Cu and W and the thermal expansion of Si and W.

KEY WORDS: absolute thermopower; electrical resistivity; heat capacity; thermal conductivity; thermal expansion; Al_2O_3; Cu; Fe; Pt; Pb; W.

1. INTRODUCTION

In 1979, the CODATA (Committee on Data for Science and Technology) Task Group on Thermophysical Properties of Solids included among its activities the preparation of a set of recommended values for some important thermal and electrical properties of solid materials which are used for checking or calibrating measuring equipment. The materials (copper, tungsten, alumina, etc.) were chosen not because they might already be nationally certified reference materials (called SRMs in the U.S.A.) but

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rather because they satisfied the following criteria: (a) extensive data are available in the literature and (b) samples are readily available in stable and well characterized form.

The Report of this Task Group entitled "Thermophysical Properties of Some Key Solids" was published in 1985 by CODATA [1]. The present paper is an updated version with amendments to take account of more recent data. These recent data are chiefly from the national laboratories and deal with the heat capacity of copper [2, 3] and tungsten [4], the thermal expansion of silicon [5] and tungsten [6], and the thermopower of tungsten [7].

The names which appear in the Acknowledgments are those who were selected by the Task Group to analyze the best available data and to produce (a) tables of mean or recommended values with estimates of uncertainties, (b) plots of the deviations of the treated data from the mean values, and (c) algebraic representations where possible. In most cases, they wrote a report suitable for publication, sent this to a reviewer suggested by the Task Group and later had these reports published. The material which appeared in the CODATA Bulletin of 1985 was abbreviated from these more complete reports. Note that the selection of data for analyses was necessarily subjective and that selection criteria varied from property to property. Where sufficient sets of data were available, least-squares fitting procedures were used.

Problems of characterization and data presentation differ from property to property. For heat capacity (C_p or C_v) and coefficient of thermal expansion (α, linear; β, volume), physical and chemical purity are not of prime importance except perhaps at very low temperatures, where the lattice energy is very small. On the other hand, the transport properties (electrical resistivity, ρ; thermal conductivity, λ; thermal diffusivity, D; thermopower, S) are dominated by impurity scattering or grain size at the lowest temperatures and these can be important at intermdiate temperatures. Thus, a change in specific impurity from 1 or 2 ppm in copper may have an undetectable effect on C_p but will alter ρ by as much as a factor of two at the lowest temperatures.

2. HEAT CAPACITY

2.1. General Discussion

The heat capacity per mole at constant pressure is denoted C_p, and that at constant volume C_v. The quantity which is usually measured is $C_p = (dH/dT)_p$, where H is the enthalpy. $C_v = (dU/dT)_v$, where U is the internal energy, is of thermodynamic interest and may be calculated from C_p using

$$C_p = C_v(1 + \beta \Upsilon T) \tag{1}$$

where β is the cubic expansion coefficient and Υ is the thermal Grüneisen parameter given by $\Upsilon = \beta B_T V/C_v \equiv \beta B_S V/C_p$. Here $B_T(B_S)$ is the isothermal (adiabatic) bulk modulus and V is the molar volume. The internal energy and hence the heat capacity may contain contributions from magnetic interactions, free electrons, rotational states, phase changes, etc., in addition to the vibrational energy of the crystal lattice. For the reference solids discussed here, the vibrational energy is dominant except for the metals at low temperatures where electrons becomes significant. As mentioned in Section 1, trace impurities (<0.1 at%) are not usually significant excepting for some magnetic impurities (Cr, Mn, Fe) which may affect the heat capacity seriously at temperatures $T \ll \theta_D$ (θ_D is the Debye characteristic temperature) where the lattice energy is small.

The present tables list recommended values for C_p and C_v for pure Cu, Fe, W, and Al_2O_3. The general form of $C_p(T)$ is illustrated in Figs. 1 and 2.

2.2. Copper

Copper is readily available at a purity (99.99+%) suitable for calorimetric standard at normal temperatures. For use below 10 or 20 K, material free of transition metal impurities (99.999+%) and hydrogen is

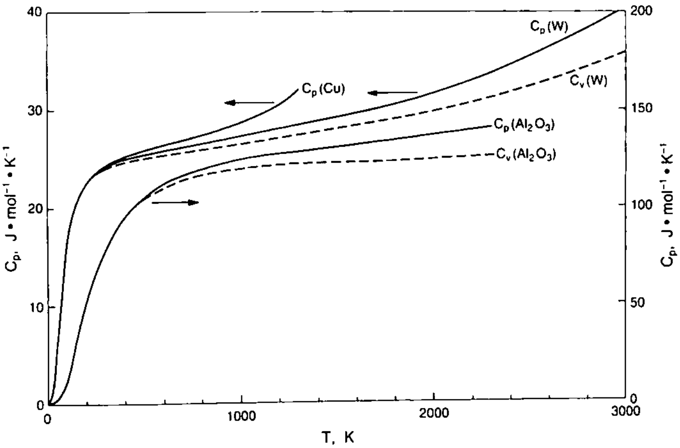

Fig. 1. Heat capacities of α-alumina, copper, and tungsten.

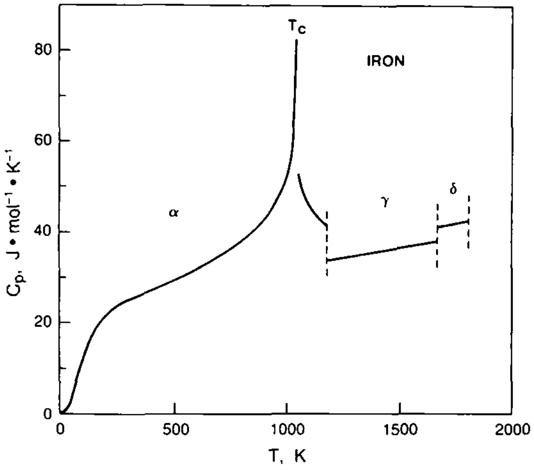

Fig. 2. Heat capacity of iron.

needed. At the 1965 Calorimetry Conference (see Ref. 8), the role of copper as a reference material for low temperatures was discussed and samples were made available for measurement in many laboratories. The values recommended for C_p in Table I are based on the following.

1 to 30 K. In 1967, Osborne et al. [8] showed that selected data from 1 to 25 K could be represented by the polynomial

$$C_p = A_1 T + A_3 T^3 \ldots A_{11} T^{11} \tag{2}$$

where C_p is in mJ · mol^{-1} · K^{-1} and T is in K.

Deviations are less than ±0.2%. A critical evaluation by Furukawa et al. [9] agreed with this.

Since that evaluation, the temperature scale has changed appreciably at low temperatures so that we now recommend (as does IUPAC [3]) the smoothed data and polynomial given by Holste et al. [10] in 1972 based on the Iowa State University Temperature Scale, which agrees more closely with the current (1990) scale. The polynomial coefficients (for C_p in mJ · mol^{-1} · K^{-1}) are (including an extra term in T^{13}) as follows.

$$A_1 = 6.9260 \times 10^{-1}$$

$$A_3 = 4.7369 \times 10^{-2}$$

$$A_5 = 1.9537 \times 10^{-6}$$

$$A_7 = 1.0869 \times 10^{-7}$$

$$A_9 = -1.9745 \times 10^{-10}$$

$$A_{11} = 1.3343 \times 10^{-13}$$

$$A_{13} = -0.32196 \times 10^{-17}$$

30 to 300 K. The 1985 CODATA values [1] were from the critical evaluation in 1967 by Furukawa et al. [9] with a polynomial fit to these by White and Collocott [11]. The IUPAC [3] used post-1967 data (e.g., Ref. 12) based on the IPTS-68 Scale to derive a polynomial fit which differs from the CODATA values by <0.3% above 50 K. Subsequently, Martin [2] published new results for the range 15 to 300 K which lie within 0.2% of the IUPAC values. For reasons of consistency, we choose to reproduce here the IUPAC polynomial $C_p = \sum A_n T^n$ J · mol^{-1} · K^{-1} (given below) and values from this are in Table I. Deviations of data for selected workers [2, 12], from the IUPAC polynomial are shown in Fig. 3.

$$A_0 = -1.285753818$$

$$A_1 = 3.098967121 \times 10^{-1}$$

$$A_2 = -2.924985792 \times 10^{-2}$$

$$A_3 = 1.418586260 \times 10^{-3}$$

$$A_4 = -3.370489513 \times 10^{-5}$$

$$A_5 = 4.856675621 \times 10^{-7}$$

$$A_6 = -4.646773402 \times 10^{-9}$$

$$A_7 = 3.070527023 \times 10^{-11}$$

$$A_8 = -1.419198886 \times 10^{-13}$$

$$A_9 = 4.557519040 \times 10^{-16}$$

$$A_{10} = -9.894731263 \times 10^{-19}$$

$$A_{11} = 1.370529662 \times 10^{-21}$$

$$A_{12} = -1.074497377 \times 10^{-24}$$

$$A_{13} = 3.517161374 \times 10^{-28}$$

300 to 1300 K. The recommended values in Table I above 300 K are those produced by Hultgren et al. [13] from selected enthalpy and heat capacity data published up to 1968. Figure 4 shows the deviation of selected sets of C_p data from those recommended. The systematic positive deviation shows that the enthalpy data used by Hultgren et al. [3] lead to smaller C_p values and suggest that the recommended values could be in error by as much as 2% above 800 K. The curve marked "Do" is from recent data measurements [4] on 99.99% pure copper [designated RM5 from the National Institute of Standards and Technology (NIST)]. The sources of the other individual curves may be found in the analysis by White and Collocott [11].

In calculating the $C_p - C_v$ correction and C_v values (Table I), expansion data are taken from Section 3.2. Values for Υ lie between 1.98 and 2.02 from 150 to 1200 K and errors in the correction factor $1 + \Upsilon\beta T = C_p/C_v$ should be $\leqslant 0.1\%$ below 100 K and reach 0.4% at 1300.

Table I. Heat Capacity of Copper

T (K)	C_p ($J \cdot mol^{-1} \cdot K^{-1}$)	C_v ($J \cdot mol^{-1} \cdot K^{-1}$)
5	0.0094	—
10	0.0554	—
20	0.462	—
25	0.957	—
30	1.688	—
35	2.628	—
40	3.725	—
45	4.92	—
50	6.16	—
55	7.41	—
60	8.62	—
65	9.79	—
70	10.89	—
75	11.91	—
80	12.87	12.82
90	14.56	14.49
100	16.00	15.90
110	17.22	17.10
120	18.26	18.11
130	19.13	18.97
140	19.86	19.64
150	20.49	20.25
160	21.03	20.76
170	21.49	21.18
180	21.90	21.55
190	22.27	21.90
200	22.58	22.18
220	23.10	22.63
240	23.53	23.00
260	23.93	23.34
280	24.22	23.58
300	24.44	23.74
350	24.88	24.02
400	25.25	24.23
450	25.61	24.44
500	25.91	24.58
550	26.21	24.70
600	26.48	24.79
650	26.73	24.87
700	26.99	24.94
800	27.48	25.03
900	28.04	25.15
1000	28.66	25.30
1100	29.48	25.57
1200	30.53	25.98
1250	31.12	26.22

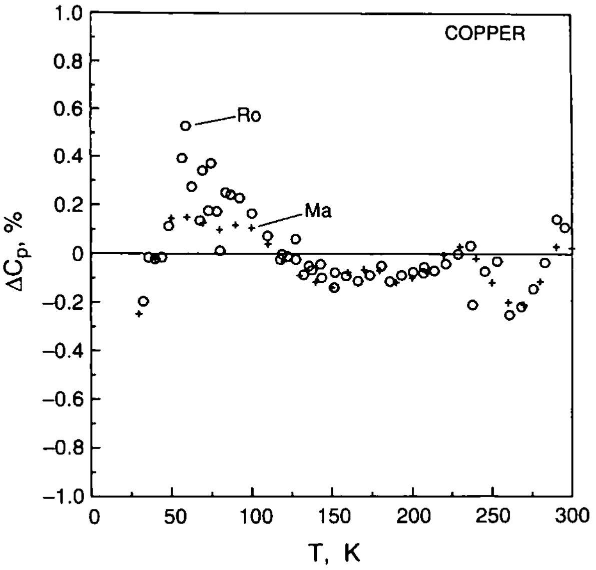

Fig. 3. Copper. Deviation of C_p data from recommended values in Table I, from 25 to 300 K.

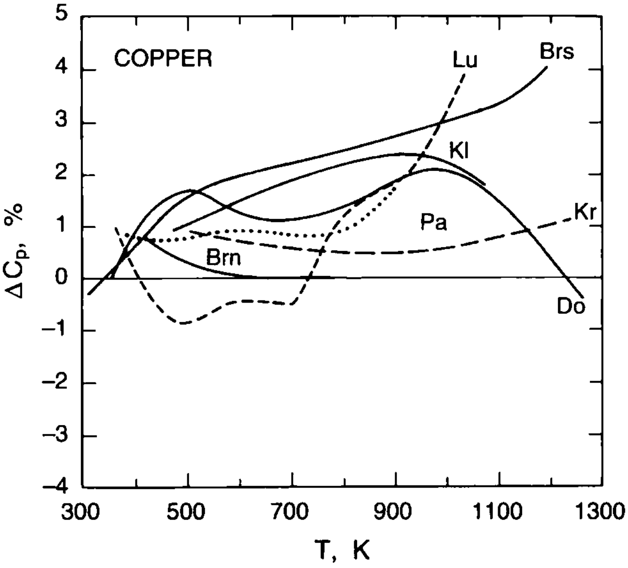

Fig. 4. Copper. Deviation of C_p data from recommended values in Table I, from 300 to 1300 K.

2.3. Iron

Iron is included because of its importance as a thermal conductivity and thermal diffusivity standard rather than for its heat capacity. The thermal conductivity λ and thermal diffusivity D are related to the heat capacity per unit volume, C_P/V, by $D=\lambda V/C_p$. Measurements and analyses of properties of iron are complicated by the ferromagnetic transition at $T_C \approx 1043$ K and crystallographic phase changes at 1185 K (α to Υ) and 1667 K (Υ to δ).

The recommended values above 10 K are those compiled by Hultgren et al. [13], which are reproduced in Table II. For $T \leqslant 10$ K, we use $C_P = 4.8 \times 10^{-3} T + 1944(T/470)^3$ J · mol^{-1} · K^{-1} from the low-temperature review of Phillips [15]. Errors may be as much as 5% from 1 to about 20 K. The deviations of selected data shown in Fig. 5 indicate that the recommended values (Hultgren et al. [13]) should not be in error by more than 1% from 50 to 800 K but deviations increase seriously above 900 K. References to data sources except for the later work of Cezairliyan and McClure [16] (denoted Ce in Fig. 5) are given by Hultgren et al. [13] and Vol. 4 of Ref. 17. The measurements were generally made on samples of "high purity" or electrolytic iron of purity >99.9%. A critical evaluation of thermodynamic data by Desai [18] recommends values for C_P from 298 to 1800 K, which differ by less than 1% from those in Table II except in the region of the transition. In the vicinity of T_C, C_P varies rapidly with T and measurements are therefore sensitive to small differences in temperature. Those who are interested in the critical region should refer to Shacklette [19].

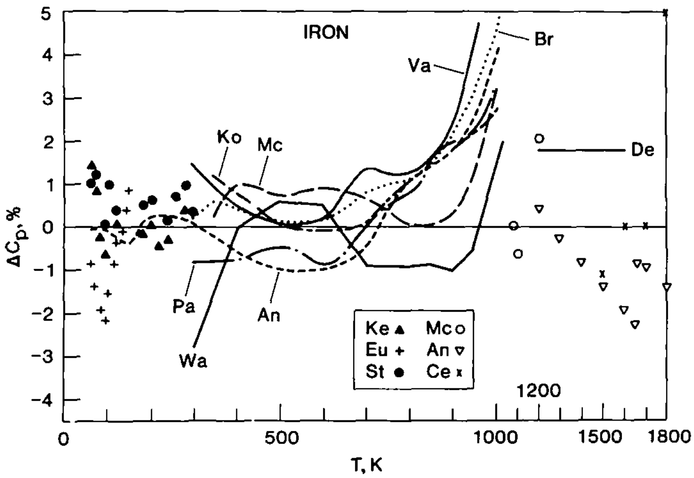

Fig. 5. Iron. Derivation of C_p data from recommended values in Table II.

Table II. Heat Capacity of Iron[a]

T (K)	C_p ($J \cdot mol^{-1} \cdot K^{-1}$)	C_v ($J \cdot mol^{-1} \cdot K^{-1}$)
5	0.028	—
10	0.068	—
15	0.140	—
20	0.25	—
25	0.44	—
30	0.74	—
35	1.15	—
40	1.66	—
45	2.29	—
50	3.01	—
60	4.76	4.76
70	6.69	6.68
80	8.64	8.63
90	10.45	10.43
100	12.05	12.01
120	14.92	14.86
140	17.18	17.09
160	19.00	18.87
180	20.32	20.15
200	21.46	22.24
200	22.50	22.24
240	23.26	22.96
260	23.85	23.51
280	24.45	24.05
300	25.02	24.58
350	26.19	25.63
400	27.36	26.67
500	29.71	28.71
600	32.05	30.73
700	34.60	32.92
800	37.95	35.97
850	40.21	38.2
900	43.10	41.0
950	47.24	45.0
1000	54.43	42.1
1020	60.0	—
1042	83.7	—
1060	51.5	—
1080	48.4	—
1100	46.40	—
1184(α)	41.42	—
1184(γ)	33.89	—
1200	34.01	—
1300	34.85	—
1400	35.69	—
1500	36.53	—
1600	37.36	—
1665(α)	37.91	—
1665(δ)	41.31	—
1700	41.46	—

[a] C_V values are not given above 1000 K for lack of information of β and B.

In calculating $C_V = C_P/(1 + \beta \Upsilon T)$ we have used values of expansion coefficient listed in the American Institute of Physics Handbook [20] and bulk modulus B_S from the elastic moduli of Leese and Lord [21]. Υ values range from 1.67 ± 0.03 from 100 to 500 K and fall to 1.4 at 800 K.

2.4. Tungsten

The CODATA recommended values prepared by White and Collocott [11] have been amended slightly at high temperatures with the incorporation of recent data measured from 1500 to 3600 K by Rignini et al. [4]. Thus, values in Table III were obtained as follows.

0 to 25 K. An unpublished analysis by Reilly and Furukawa [22] and more recent data [11] indicate that $C_p = 1.01T + 0.0346T^3 + 2.84 \times 10^{-8}T^7$ $\text{mJ} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ over this range, with a possible error of 10% below 5 K, decreasing to 2% at 20 K.

25 to 300 K. Selected values are from analysis by Reilly and Furukawa [22] and differ from those of Hultgren et al. [13] by less than 1% over most of this range. Figure 6 shows the deviation of the few sets of input data. We have fitted a polynomial, $C_p = \Sigma A_n(T/100)^n$ (where C_p is in $\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ and T is in K) to the selected data with an rms deviation of 0.25%, the maximum deviation being 1% between 25 and 30 K. The coefficients are as follows.

$A_0 = 7.82880$ $A_3 = -4.636715 \times 10^2$ $A_6 = 5.449818 \times 10$

$A_1 = -8.395318 \times 10$ $A_4 = 3.775531 \times 10^2$ $A_7 = -8.85386$

$A_2 = 3.173221 \times 10^2$ $A_5 = -1.852978 \times 10^2$ $A_8 = 6.10992 \times 10^{-1}$

300 to 3400 K. Eleven selected sets of data were fitted to a polynomial over the range of 300 to 3400 K with an rms deviation of 1.1%. The deviation of these data from the mean are shown in Fig. 7 to be generally less than 1% below 1000 K and less than 2.5% above this. The recent data from Righini et al. are marked RSBR. Sources of the other curves may be found in the earlier analysis of White and Collocott [11]. The polynomial fit for this range is $C_p = \Sigma A_n(T/1000)^n$, where

$A_{-1} = 1.406637 \times 10^1$ $A_0 = 2.1868372 \times 10^{-1}$

$A_1 = 8.068661$ $A_2 = -3.756196$ $A_3 = 1.075862$

C_p is in $\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ and T is in K.

Table III. Heat Capacity of Tungsten

T (K)	C_P ($J \cdot mol^{-1} \cdot K^{-1}$)	C_V ($J \cdot mol^{-1} \cdot K^{-1}$)
10	0.045	—
15	0.132	—
20	0.333	—
25	0.739	—
30	1.35	—
35	2.22	—
40	3.30	—
50	5.82	—
60	8.39	—
70	10.74	—
80	12.81	—
90	14.57	14.57
100	16.04	16.02
120	18.28	18.24
140	19.87	19.82
160	21.01	20.95
180	21.86	21.78
200	22.51	22.41
250	23.65	23.52
273.15	23.98	23.84
300	24.35	24.20
350	24.69	24.52
400	24.92	24.74
500	25.38	25.16
600	25.83	25.51
700	26.25	25.86
800	26.64	26.14
900	27.03	26.45
1000	27.39	26.70
1100	27.76	27.00
1200	28.12	27.25
1300	28.48	27.54
1400	28.85	27.82
1500	29.24	28.12
1600	29.67	28.42
1700	30.10	28.72
1800	30.57	29.0
2000	31.66	29.8
2200	32.96	30.7
2400	34.53	31.8
2600	36.42	33.1
2800	38.68	34.6
3000	41.36	36.4
3200	44.52	38.0
3400	48.21	39.9

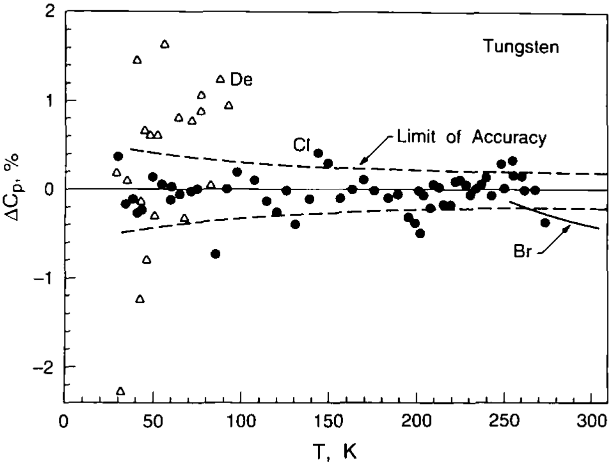

Fig. 6. Tungsten. Deviation of C_p data from recommended values in Table III, from 30 to 300 K.

In calculating the C_V values, expansivity data were taken from Section 3.4. Values for Υ lie between 1.58 and 1.62 from 100 to 1600 K, increasing gradually to 1.73 at 3000 K. Possible errors introduced in this correction from C_P to C_V should be $<0.2\%$ below 2000 K and 0.6% at 3000 K.

2.5. Alumina (Sapphire, Corundum)

This material was proposed as a calorimetric standard at the 1948 Calorimetry Conference [23] and then became available as an NBS Standard Reference Material in 1970 [24]. Therefore, it has been a popular material for measurement, usually in the form of synthetic sapphire.

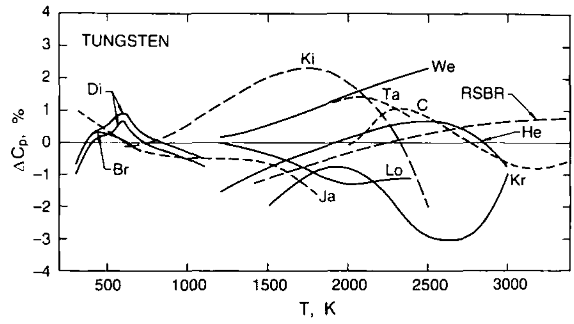

Fig. 7. Tungsten. Deviation of C_p data from recommended values in Table III, from 300 to 3400 K.

Table IV. Heat Capacity of Alumina

T (K)	C_P ($J \cdot mol^{-1} \cdot K^{-1}$)	C_V ($J \cdot mol^{-1} \cdot K^{-1}$)
10	0.0087	—
20	0.073	—
25	0.145	—
30	0.263	—
35	0.443	—
40	0.698	—
50	1.506	—
60	2.793	—
70	4.594	—
80	6.902	—
90	9.677	—
100	12.85	12.85
120	20.07	20.06
140	27.94	27.91
160	36.04	35.93
180	43.91	43.72
200	51.33	51.09
250	67.17	66.82
273.15	73.22	72.85
300	79.64	78.95
350	88.91	88.16
400	96.08	95.03
500	106.13	104.56
600	112.55	110.45
700	116.92	114.3
800	120.14	116.9
900	122.81	119.0
1000	125.02	120.6
1100	126.79	121.7
1200	128.25	122.5
1300	129.22	122.8
1400	130.16	123.0
1500	131.0	123.1
1600	132.1	123.5
1700	133.2	123.9
1800	134.3	124.3
2000	137.0	125.2
2200	139.7	126.2
2300	141.3	126.8

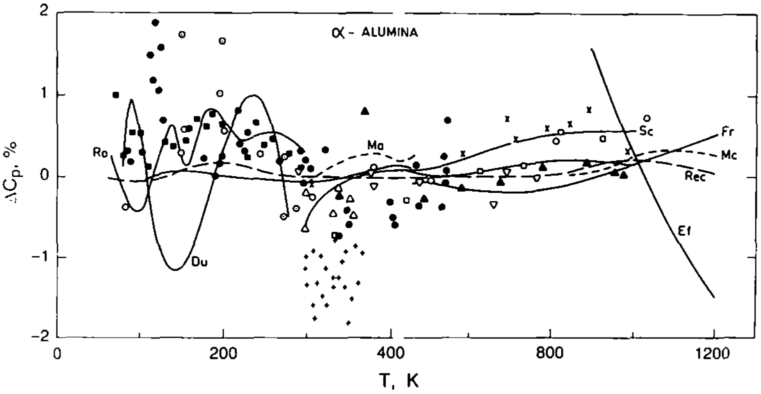

Fig. 8. α-Alumina. Deviation of C_p data (including recommended values, "Rec") from baseline of Castanet [25] from 70 to 1200 K (note change of scale at 400 K).

Castanet [25] has selected the "best" (internally consistent) sets of data of C_P and averaged them to produce the recommended values of Table IV. At high temperatures, Castanet included five sets of enthalpy (H_T–H_{298}) measurements because of a shortage of C_P data. Many sets of measurements done at the NBS (National Bureau of Standards, now National Institute of Standards and Technology or NIST) since 1950 have been resmoothed by Ditmars et al. [24] and these have been used as a baseline by Castanet in preparing the deviation graphs (Figs. 8 and 9). These indicate that the

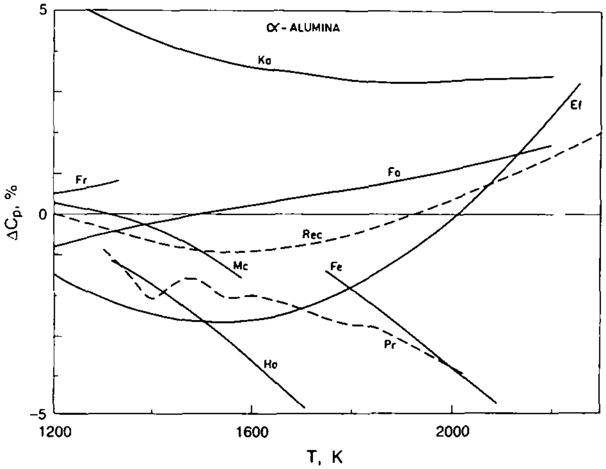

Fig. 9. α-Alumina. Deviation of C_p data (including recommended values, "Rec") from baseline of Castanet [25] from 1200 to 2500 K.

presently recommended values do not differ from the NBS baseline by more than 0.5% from 20 to 1200 K or more than 1% from 1200 to 2000 K. The sources of the individual curves and points in Figs. 8 and 9 may be found in the detailed analysis of Castanet [25].

Note that IUPAC also published in 1987 [3] an evaluation for "Recommended Reference Materials" including C_p for α-alumina. Their values agree closely with the CODATA (and present) values below 1300 K. The difference (IUPAC-CODATA) reaches +1% at 1600 K, then changing to −1.4% at 2200 K.

For convenience of data users, Castanet [25] has fitted polynomials over three ranges to the recommended values with deviations which are <0.2% above 40 K. Note that these polynomials do not join smoothly from one range to another. At the lowest temperatures ($0 < T < 20$ K), $C_P = (8.8 \pm 0.1) \times 10^{-6} T^3$ J·mol^{-1}·K^{-1}, which gives a limiting value for the Debye temperature of $\theta_0 = 1035$ K, similar to that calculated from the elastic moduli.

Coefficients for polynomail $C_p = \Sigma A_n T^n$ for α-alumina are as follows:

	20 to 150 K	160 to 1000 K	1000 to 2250 K
A_{-3}	-1.0315×10^3	-1.68786×10^8	—
A_{-2}	2.46106×10^3	5.16926×10^6	-6.6219724×10^7
A_{-1}	-3.50560×10^2	-5.9188×10^4	1.492792×10^5
A_0	1.96082×10^1	2.71013×10^2	—
A_1	-5.22293×10^{-1}	-1.93999×10^{-1}	4.45658×10^{-2}
A_2	6.35986×10^{-3}	1.463859×10^{-4}	-2.570264×10^{-6}
A_3	-1.48727×10^{-5}	-5.097507×10^{-8}	—
A_4	—	6.53021×10^{-12}	—

where C_p is in J·mol^{-1}·K^{-1} and T is in K.

To calculate the $C_p - C_v$ correction and C_v values (Table IV), expansion data are taken from Section 3.5. Values for the Grüneisen parameter Υ increase slowly with temperature from 1.30 at 250 K to 1.35 at 1000 K and 1.40 at 2000 K. Errors in the correction factors $(1 + \beta \Upsilon T)$ should be less than 0.2% even at the highest temperature, which is much less than likely errors in choosing C_p.

3. THERMAL EXPANSION

3.1. General Discussion

The linear coefficient of expansion, α, is defined thermodynamically as

$$\alpha = \left(\frac{d \ln l}{dT}\right) = \frac{1}{l_T}\left(\frac{dl}{dT}\right) \tag{3}$$

For technical purposes the more convenient coefficient is

$$\alpha^* = \frac{1}{l_{293}}\left(\frac{dl}{dT}\right) \tag{4}$$

where l_{293} is the length measured at 20°C (293.15 K). Also useful is the integrated expansion

$$\frac{\Delta l}{l_{293}} = \frac{l_T - l_{293}}{l_{293}} \tag{5}$$

which can be used to convert α to α^*:

$$\alpha = \alpha^* \left(1 + \frac{\Delta l}{l_{293}}\right)^{-1} \tag{6}$$

The present tables list our recommended values for α^* and $\Delta l/l_{293}$ for copper, silicon, tungsten, and α-alumina. Values for the volume coefficient, β, may be calculated for the cubic elements (Cu, Si, W) simply as $\beta = 3\alpha$ or, for anisotropic (but axially symmetric) sapphire, as $\beta = 2\alpha_\perp + \alpha_\parallel$, where $\alpha_\perp$ and $\alpha_\parallel$ are, respectively, the linear coefficients measured normal and parallel to the symmetry axis.

The recommended values have been obtained from a least-squares fit to selected sets of experimental data, as discussed in more detail in Ref. 26. Most sets of data for Δl or α^* are given in Vols. 12 and 13 of Ref. 17 with original references. Variations of α^* among some observers may exceed 10 or 20% and usually arise from measurement errors rather than from differences in materials: expansion (like heat capacity) depends primarily on the host lattice and not on trace impurities, except at the lowest temperature ($T \ll \theta_D$, Debye characteristic temperature). A useful guide to the reliability of data is the constancy of the ratio of expansion coefficient to heat capacity or the Grüneisen ratio $\Upsilon = \beta V B_S / C_P$, which is a measure of the lattice anharmonicity and is fairly constant at $T \geqslant \theta_D/2$. This fact can help in interpolating or rejecting data. Algebraic expressions for α^* are also given for normal and higher temperatures. At low temperatures where a number of overlapping polynomials are needed, we refer to fuller treatments of the data.

3.2. Copper

The magnitude of $\alpha^*(T)$ is illustrated in Fig. 10 and recommended values are in Table V. These are taken from the analyses by White and

Table V. Values of α^* and $\Delta l/l_{293}$ for Copper[a]

T (K)	α^* ($10^{-6}\,K^{-1}$)	$\Delta l/l_{293}$ (10^{-6})
10	0.030	−3257
12	0.052	−3257
14	0.083	−3257
16	0.126	−3257
18	0.186	−3256
20	0.263	−3256
25	0.56	−3254
30	1.00	−3250
35	1.58	−3244
40	2.27	−3234
50	3.84	−3204
60	5.46	−3157
70	6.98	−3095
80	8.33	−3018
90	9.49	−2929
100	10.49	−2829
120	12.05	−2602
140	13.19	−2349
160	14.03	−2077
180	14.67	−1789
200	15.19	−1491
250	16.11	−706
293.15	16.65	0
300	16.70	114
350	17.12	959
400	17.51	1824
500	18.23	3611
600	18.93	5469
700	19.67	7398
800	20.46	9404
900	21.32	11490
1000	22.26	13670
1100	23.31	15950
1200	24.58	18340

[a] Maximum probable errors in $10^6\alpha^*$ (K^{-1}) are 0.001 (below 20 K), 0.02 (40–300 K), 0.05 (350–800 K), and 0.1 (above 800 K).

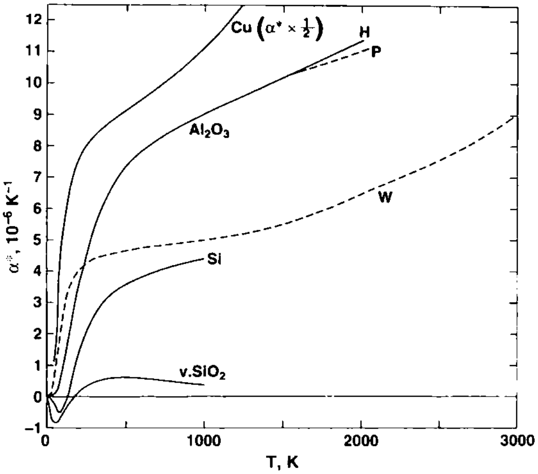

Fig. 10. $\alpha^* = (1/l_{293})(dl/dT)$ for copper, silicon, tungsten, and α-alumina ($\alpha^*_{av} \simeq \alpha^*_{55^\circ}$). H and P denote Hahn and Petukhov, respectively [35].

Roberts [26] of eight selected sets of data for the range 300 to 1250 K and from Kroeger and Swenson [27] below 300 K. Original data were obtained on copper of purity ⩾99.98%, for which there is no evidence of measurable changes in α due to impurity above 20 K. Deviations of selected data are shown above 50 K in Figs. 11 and 12. Sources of the individual curves and points in Figs. 11 and 12 may be found in the fuller analyses [26, 27]. Polynomial expressions for $\alpha^*(T)$ are given by Kroeger

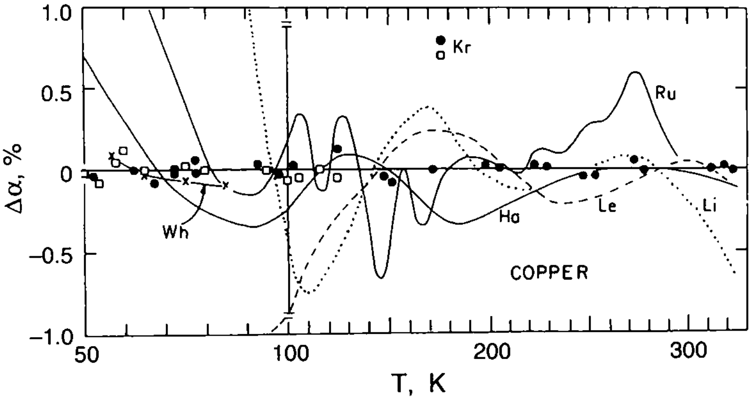

Fig. 11. Copper. Deviation of α data from recommended values in Table V, from 50 to 300 K [27].

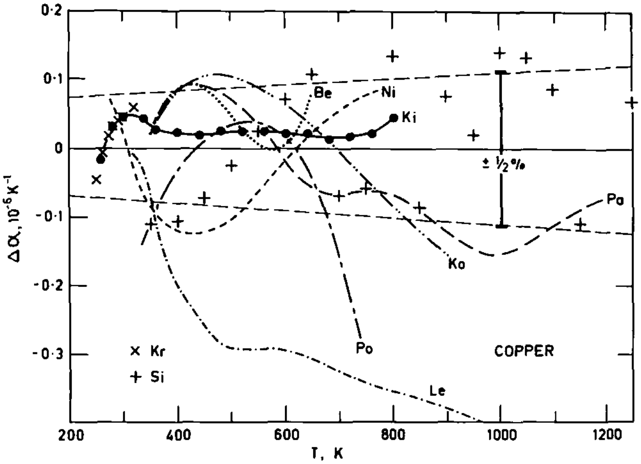

Fig. 12. Copper. Deviation of α data from recommended values in Table V, from 300 to 1200 K [26].

and Swenson [27] for overlapping ranges 2 to 30, 20 to 55, 45 to 110, and 90 to 325 K. For the range 325 to 1250 K,

$$\alpha^* = 11.1113 + 33.360(T/1000) - 74.022(T/1000)^2 + 99.728(T/1000)^3 - 64.930(T/1000)^4 + 17.015(T/1000)^5 \tag{7}$$

where α^* is in $10^{-6}\,K^{-1}$ and T is in K.

3.3. Silicon

Swenson published, in 1983 [28], a detailed analysis of the linear expansion data for silicon using three sources of data below 300 K and four sets from 300 to 800 K. More recently, measurements on high-purity silicon at the National Research Laboratory of Metrology (NRLM) in Japan [5] have extended the range of accurate data up to 1300 K. Swenson [29] has included these in a reanalysis fitting from 90 to 1300 K to the expression.

$$\alpha^* = 0.713 + 5.04\frac{x^2e^x}{(e^x - 1)^2} + \frac{0.180(y-1)^2}{(1 + 0.360y)} \tag{8}$$

where α^* is in $10^{-6}\,K^{-1}$, $x = 685/T$, $y = T/435$, and T is in K.

The recommended values from this fit are given in Table VI and deviations of the source data (Fig. 13) from the fit are within ±1%, above 200 K. The designations of the individual curves shown in Fig. 13 (from the

Table VI. Values of α^* and $\Delta l/l_{293}$ for Silicon[a]

T (K)	α^* (10^{-6} K^{-1})	$\Delta l/l_{293}$ (10^{-6})
10	0.0005	−215.5
12	0.0008	−215.5
14	0.0013	−215.5
16	0.0011	−215.5
18	−0.0001	−215.5
20	−0.003	−215.5
25	−0.019	−216
30	−0.053	−216
35	−0.103	−216
40	−0.164	−217
50	−0.29	−219
60	−0.40	−223
70	−0.46	−227
80	−0.47	−232
90	−0.43	−236
100	−0.34	−240
120	−0.06	−244.5
140	0.31	−242
160	0.69	−232
180	1.06	−214.5
200	1.40	−189.5
250	2.10	−101
293.15	2.56	0
300	2.62	18.0
350	2.99	158.5
400	3.26	315
500	3.61	660
600	3.83	1033
700	4.00	1424
800	4.11	1829
900	4.21	2244
1000	4.30	2670
1100	4.39	3105
1200	4.47	3547
1300	4.56	3999

[a] Maximum probable errors in $10^6\alpha^*$ (K^{-1}) are 0.001 (below 20 K), 0.02 (40–600 K), and 0.05 (600–1200 K).

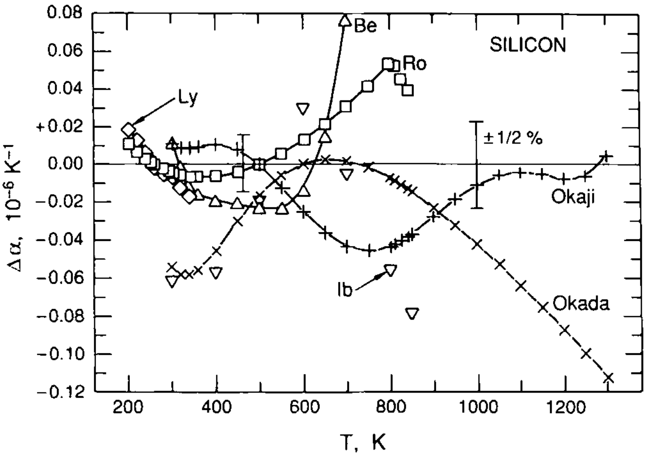

Fig. 13. Silicon. Deviation of α data from recommended values in Table VI [28, 29].

unpublished analysis of Swenson [29]) correspond to the following sources: Okaji [5], Okada [30], Ly [31], Be [32], Ib [33], and Ro [34].

3.4. Tungsten

Values in Table VII are an amended version of the analysis by White and Roberts [35] incorporating recent data from 1500 to 3600 K from the NBS [6] (now NIST) in addition to the 10 sets of data above 300 K and 3 sets below 300 K used previously. The tungsten specimens varied from sintered and arc-cast rod to thin wire filaments. The data above 300 K are a least-squares fit to the expression,

$$\alpha^* = 3.873 + 2.562(T/1000) - 2.8613(T/1000)^2 + 1.9862(T/1000)^3 - 58608(T/1000)^4 + 0.070586(T/1000)^5 \qquad (9)$$

where α^* is in $10^{-6}\,\mathrm{K}^{-1}$ and T is in K.

Deviations are shown in Fig. 14. The sources of individual curves are identified in the analysis in Ref. 35 with the addition of “Mi” from Miiller and Cezairliyan [6] and Curve 85, which is the difference between the CODATA recommendation in 1985 and the present Table VII.

3.5. α-Alumina (Corundum, Sapphire)

The expansion of single crystals of alumina is anisotropic, the coefficient parallel to the symmetry axis ($\alpha_{\parallel}$) being ~10% larger than that

Table VII. Values of α^* and $\Delta l/l_{293}$ for Tungsten[a]

T (K)	α^* (10^{-6} K^{-1})	$\Delta l/l_{293}$ (10^{-6})
10	0.006	−876
15	0.019	−876
20	0.048	−876
25	0.102	−875
30	0.20	−875
40	0.53	−871
50	0.96	−864
60	1.43	−852
70	1.88	−8.35
80	2.30	−814
90	2.61	−790
100	2.88	−762
120	3.30	−700
140	3.59	−631
160	3.81	−557
180	3.97	−479
200	4.10	−398
250	4.30	−188
293.15	4.42	0
300	4.43	31
350	4.48	254
400	4.55	479
500	4.65	940
600	4.74	1410
700	4.82	1890
800	4.89	2375
900	4.97	2865
1000	5.05	3365
1100	5.13	3870
1200	5.22	4390
1300	5.32	4910
1400	5.43	5450
1500	5.55	6000
1600	5.68	6570
1700	5.83	7160
1800	5.98	7760
2000	6.32	9020
2200	6.72	10360
2400	7.18	11780
2600	7.71	13280
2800	8.34	14890
3000	9.12	16620
3200	10.09	18520
3400	11.33	20680
3500	12.07	

[a] Maximum probable errors in $10^6\alpha^*$ (K^{-1}) are 0.001 (below 20 K), 0.01 (20–40 K), 0.05 (40–300 K), 0.01 (300–1800 K), and 0.2 (above 1800 K), and that in $10^6 \Delta l/l$ is 0.5% (below 3000 K).

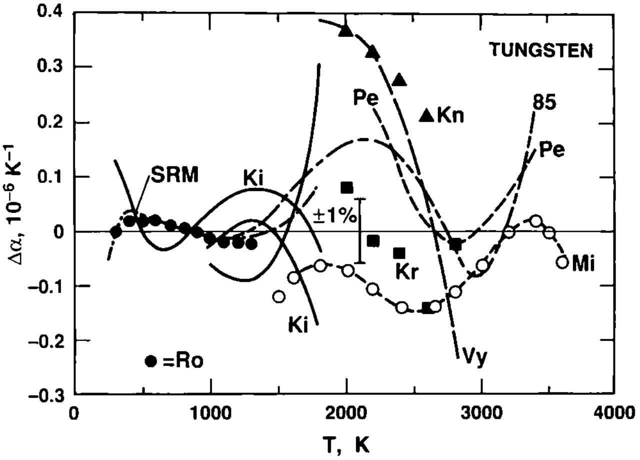

Fig. 14. Tungsten. Deviation of α data from recommended values in Table VII [35, 5].

normal to this axis ($\alpha_\perp$) over a wide temperature range. The average value for a polycrystal without preferred orientation is $\alpha_{av} = \beta/3 = (2\alpha_\perp + \alpha_\parallel)/3$ and should equal α measured on a crystal in a direction inclined at an angle $\psi \approx 55°$ to the symmetry axis. White and Roberts [35] have taken data from seven sources obtained on either polycrystals or single crystals

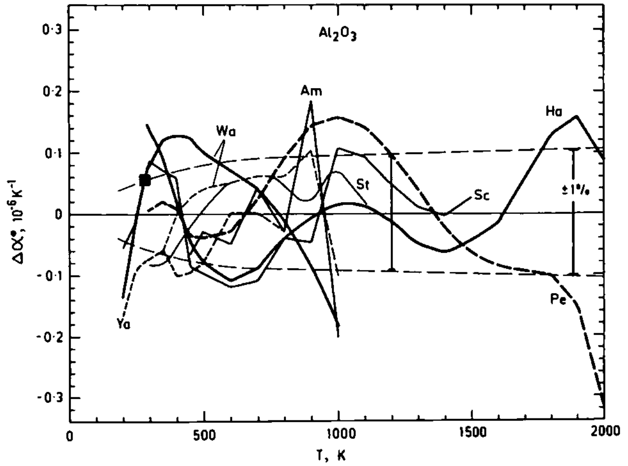

Fig. 15. Alumina. Deviation plot of the average linear coefficients, α_{av}, from recommended values in Table VIII [35].

($\|$, $\perp$, and for $\psi = 55$–$59°$) at temperatures between 300 and 2000 K. These were fitted to the following fifth-order polynomial:

$$\alpha^* = -3.26195 + 46.70597(T/1000) - 76.71389(T/1000)^2 + 64.44551(T/1000)^3 - 26.19865(T/1000)^4 + 4.111538(T/1000)^5 \qquad (10)$$

where α^* is in $10^{-6}\,\mathrm{K}^{-1}$ and T is in K.

The mean values are listed in Table VIII and deviations are shown in Fig. 15. The values of α_{av} below 300 K are taken from more limited data and were smoothed with the aid of the Grüneisen relation but could not

Table VIII. Values of α^* and $\Delta l/l_{293}$ for Alumina[a]

T (K)	$\alpha^*_{\|}$ ($10^{-6}\,\mathrm{K}^{-1}$)	$\alpha^*_{\perp}$ ($10^{-6}\,\mathrm{K}^{-1}$)	α^*_{av} ($10^{-6}\,\mathrm{K}^{-1}$)	$\Delta l/l_{293}$ (10^{-6})	T (K)	α^*_{av} ($10^{-6}\,\mathrm{K}^{-1}$)	$\Delta l/l_{293}$ (10^{-6})
20	0.006	0.003	0.004	−635	1,100	9.34	6,485
30	0.022	0.013	0.016	−635	1,200	9.59	7,430
40	0.054	0.039	0.044	−635	1,300	9.85	8,400
50	0.120	0.08	0.095	−634	1,400	10.09	9,400
60	0.22	0.16	0.18	−633	1,500	10.31	10,420
70	0.35	0.26	0.29	−631	1,600	10.51	11,460
80	0.53	0.40	0.44	−627	1,700	10.67	12,520
90	0.73	0.57	0.61	−622	1,800	10.84	13,600
100	0.98	0.73	0.81	−615	1,900	11.05	14,690
120	1.52	1.15	1.28	−595	2,000	11.37	15,810
140	2.14	1.64	1.80	−565			
160	2.72	2.15	2.34	−524			
180	3.30	2.70	2.90	−473			
200	3.84	3.21	3.42	−410			
250	5.01	4.27	4.52	−212			
293	5.80	5.06	5.30	0			
300	5.90	5.15	5.40	37			
350	6.50	5.77	6.08	324			
400	7.08	6.36	6.64	642			
450	7.57	6.81	7.10	986			
500	7.94	7.19	7.46	1,350			
550	8.30	7.50	7.75	1,730			
600	8.55	7.72	7.99	2,125			
650	8.79	7.93	8.18	2,530			
700	8.97	8.10	8.35	2,940			
800	9.29	8.36	8.62	3,790			
900	9.54	8.65	8.86	4,665			
1,000	9.70	8.83	9.09	5,560			

[a] Maximum probable errors in $10^6\,\alpha^*$ (K^{-1}) are 0.01 (below 50 K), 0.03 (100–300 K), and 0.1 (300–2000 K).

be fitted with the same polynomial expression. Values of $\Delta l/l_{293}$ were integrated from α^*_{av}.

Values of $\alpha^*_{\parallel}$ and $\alpha^*_{\perp}$ were not fitted to a polynomial because of paucity of data, there being only four sets from to 1000 K and only one above 1000 K. There are three below 300 K which differ significantly. Note that in the table, there is not exact consistency at all temperatures between values of α^*_{av} and the single-crystal average $(\alpha^*_{\parallel} + 2\alpha^*_{\perp})/3$. This is because α^*_{av} is based on more data sets, i.e., includes polycrystals, than $\alpha_{\parallel}$ and $\alpha_{\perp}$. The difference is within the probable limits of error, namely, 0.1×10^{-6} from 200 to 1000 K and $0.02 \times 10^{-6}\ \mathrm{K}^{-1}$ below 100 K.

4. ELECTRICAL RESISTIVITY

4.1. General Discussion

The electrical resistivity, ρ, of a rod of length l, area A, and resistance R at temperature T are related by $\rho(T) = R(T)(A/l)_T$. If A and l are measured at room temperature (say 293 K), then

$$\rho^*(T) = R(T)(A/l)_{293} \tag{11}$$

$$\rho(T) \simeq \rho^*(T)(1 + \Delta l(T)/l_{293}) \tag{12}$$

Values of ρ tabulated below have been corrected for the expansion $\Delta l/l$.

Like other transport properties, the electrical resistivity is determined by the number and velocity of available carriers (of electric charge) and by the scattering of these carriers. In metallic elements at high temperatures, scattering is produced predominantly by a contribution, ρ_i, from thermal vibrations and at lower temperatures by a contribution, ρ_0, from static imperfections in the crystal lattice. These contributions are not strictly additive, so that $\rho = \rho_0 + \rho_i + \Delta$, where the deviation Δ depends on the temperature, ρ_0, and the nature of the imperfections.

At high temperatures, $T > \Theta_{\mathrm{D}}$ (where the characteristic temperature Θ_{D} lies in the range 200 to 400 K for most metals), accurate measurement on many high-purity specimens of a given metallic element confirm that the ideal resistivity, ρ_i, is an intrinsic property of the electron–lattice system for that element so that we can recommend values for this parameter. However, at very low temperatures $(T \ll \Theta_{\mathrm{D}})$, the defect resistivity, ρ_0, predominates, is characteristic only of a particular piece of mtal, and is, therefore, not of general usefulness. At intermediate temperatures, the reliability of estimates of $\rho(T)$ depends on the relative magnitudes of ρ_i and ρ_0. We characterize the electrical purity of a metal by the residual resistance ratio, $\mathrm{RRR} = R_{273}/R_{4.2}$ (the ratio of resistance at the ice point to resistance at the boiling point of liquid helium) or, occasionally, the ratio $R_{293}/R_{4.2}$. If this ratio is large, say 1000, then the intrinsic scattering ρ_i will predominate down to temperatures $T \sim \Theta_D/10$.

We recommend that the appropriate value of ρ at temperature T above $\Theta_D/10$ be given by

$$\rho \simeq \rho_i(\text{tabulated}) + \rho_0 \tag{13}$$

where ρ_0 is measured in liquid helium for the particular material and deviations Δ are neglected. This is most reliable for an element such as platinum of thermometer-grade, for which there are many sets of data on wires of comparable purity, e.g., RRR of 1500 to 3000. It will be less reliable for "pure" copper, for which RRR may vary from 50 to 50,000. Figure 16 shows how ρ varies with temperature for the four recommended elements.

4.2. Copper

There are nearly 300 sets of experimental data reporting the electrical resistivity of copper as a function of temperature. Matula of CINDAS [36] has evaluated these and produced the recommended value shown in Table IX. These are based primarily on three sets of data, two of which were also chosen by Bass [37] for the tabulation in Landolt–Börnstein. These data, originating in the National Research Council of Canada, the Oak Ridge National Laboratory, and the National Bureau of Standards

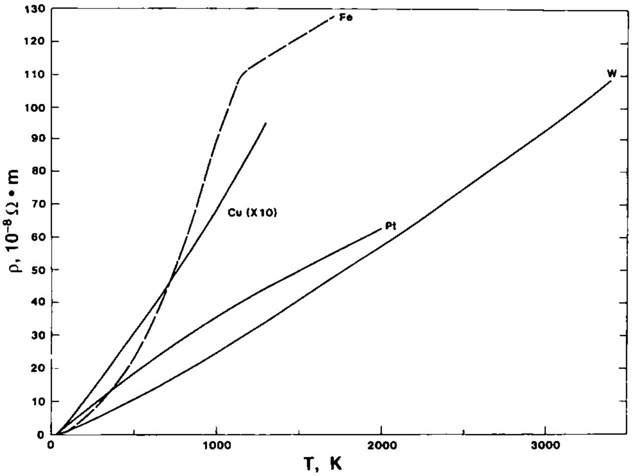

Fig. 16. Copper, iron, platinum, and tungsten. Variation of $\rho_i(T)$ corrected for thermal expansion.

(now National Institute for Standards and Technology), were fitted to a modified Grüneisen–Bloch relation, i.e., derived from the form

$$F(x) = 4\left(\frac{T}{\Theta}\right)^5 \int_0^{\Theta/T} \frac{e^x x^5}{(e^x - 1)^2}\,dx \tag{14}$$

where $x = \Theta/T$ and Θ is a characteristic temperature.

The input data were for specimens of annealed 99.999% pure copper having RRR values of ~ 1000. Matula tabulates both total resistivity, ρ, and ideal resistivity, $\rho_i = \rho - \rho_0$, after correction for thermal expansion. His estimate of the uncertainty in ρ_i is 10% at 20 K, 4% at 30 K, 3% from 30 to 100 K, 1% from 100 to 250 K, 0.5% from 250 to 350 K, 1% from 350 to 500 K, and 4% above 500 K.

Figure 17 shows the differences of selected data from recommended values. The sources of the individual curves are given in the compilation by Bass [37]. A fifth-order polynomial has been fitted to the corrected recommended values of ρ_i (Table IX) from 100 to 1300 K with deviations of less than 0.1% except below 200 K, where the deviation reaches 0.4% near 125 K.

The polynomial is $\rho = \sum A_n (T/1000)^n$, where

$$A_0 = -3.843 \times 10^{-1}$$
$$A_1 = 7.5319$$
$$A_2 = -2.6988$$
$$A_3 = 3.9604$$
$$A_4 = -2.1106$$
$$A_5 = 5.579 \times 10^{-1}$$

and ρ is in $10^{-8}\,\Omega \cdot \mathrm{m}$.

4.3. Iron

The recommended values for ρ_i (Table IX) were prepared by Chu and Ho [38] and are corrected for thermal expansion. Their report states that the "values above 200 K are for iron of purity 99.99% or higher, while those below 200 K are applicable only to highly purified zone—refined iron having a residual resistivity of $0.02 \times 10^{-8}\,\Omega \cdot \mathrm{m}$... ." The estimated uncertainty is ±5% below 100 K, ±3% from 100 to 200 K, and ±2% from

Table IX. Values of the Ideal Electrical Resistivity, ρ_i, of Copper and Iron (Corrected for Expansion)[a]

T (K)	ρ_i(Cu) ($10^{-8}\ \Omega \cdot \mathrm{m}$)	ρ_i(Fe) ($10^{-8}\ \Omega \cdot \mathrm{m}$)
20	0.00080	0.0061
22	0.00129	0.0080
24	0.00202	0.0101
26	0.00304	0.0128
28	0.00442	0.0158
30	0.00628	0.0195
35	0.0127	0.0329
40	0.0219	0.0532
45	0.0338	0.0832
50	0.0498	0.125
55	0.0707	0.179
60	0.0951	0.247
65	0.123	0.331
70	0.152	0.428
75	0.182	0.540
80	0.213	0.669
85	0.246	0.798
90	0.279	0.942
100	0.346	1.26
120	0.485	1.97
140	0.626	2.74
160	0.767	3.54
180	0.906	4.36
200	1.044	5.17
220	1.181	6.04
240	1.317	6.95
260	1.453	7.90
273.15	1.541	8.55
293.15	1.676	9.59
300	1.723	9.96
350	2.061	12.85
400	2.400	16.08
450	2.742	19.69
500	3.088	23.70
550	3.437	28.15
600	3.790	33.04
650	4.147	38.40
700	4.512	44.25

[a] $\rho = \rho_0 + \rho_i$.

Table IX. *(Continued)*

T (K)	ρ_i (Cu) ($10^{-8}\,\Omega\cdot\mathrm{m}$)	ρ_i (Fe) ($10^{-8}\,\Omega\cdot\mathrm{m}$)
750	4.882	50.62
800	5.260	57.54
850	5.645	65.00
900	6.039	73.09
950	6.443	81.94
1000	6.856	91.74
1050	7.279	102.5
1100	7.715	108.3
1150	8.163	111.5
1200	8.624	112.6
1250	9.100	114.5
1300	9.590	116.4
1400		119.7
1500		122.8
1600		125.8
1700		128.0

200 K up to the melting point. Deviations of some data sets from the recommended values are shown in Fig. 18, the selection being those sets of data preferred in Ref. 38 or by Bass [37]. Original sources of the individual sets are identified in Refs. 37 and 38. Because $\rho(T)$ is rather complex for iron above 1000 K due to phase transitions, no single algebraic expression fits the data well. Chu and Ho [38] give polynomials for eight ranges to

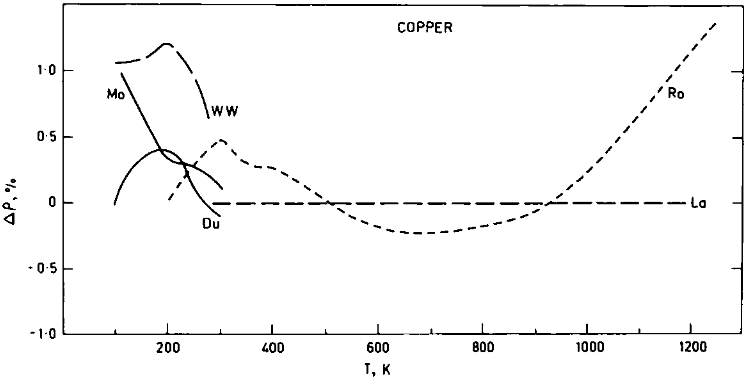

Fig. 17. Copper. Percentage deviation of selected ρ_i values from recommended values (Table X) above 100 K.

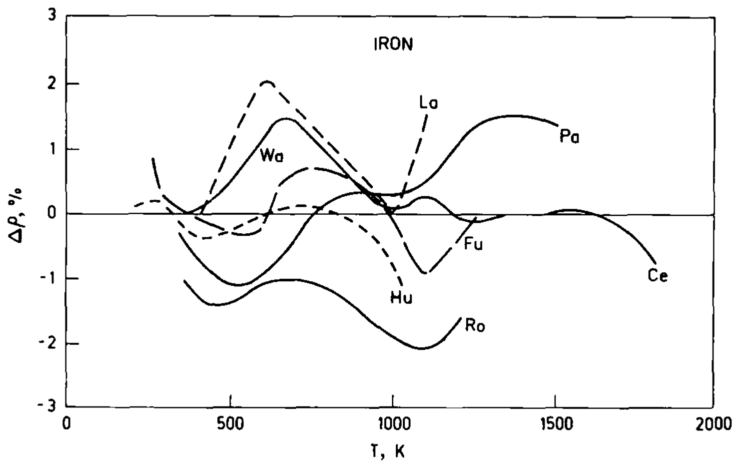

Fig. 18. Iron. Percentage deviation of selected ρ_i values from recommended values (Table IX) above 100 K.

represent values which are not corrected (for expansion). For the region below 200 K,

$$\rho_i = 58.1\left(\frac{T}{467}\right)^5 \int_0^{467/T} \frac{e^x x^5}{(e^x - 1)^2}\,dx + 1.3\times10^{-5}T^2 \qquad (15)$$

and from 200 to 900 K,

$$\rho_i = -1.1207 + 22.615(T/1000) + 39.139(T/100)^2 + 29.526(T/1000)^3 \qquad (16)$$

where ρ is in $10^{-8}\,\Omega\cdot\text{m}$ and T is in K.

4.4. Platinum

Many detailed measurements have been made of the resistance, $R(T)$, of "pure" platinum because of its use as a thermometer. The International Temperature Scale (presently called ITS-90) is defined from 13.8 to 1235 K in terms of the resistance ratio, $W_T = R_T/R_{273}$, of suitably pure platinum, using polynomials to express $W(T)$. There have also been efforts to extend the use of platinum thermometers to the melting point of gold (1337.6 K). White [39] has used these resistance data, corrected where necessary for impurity scattering, converted them to resistivities by assuming the resistivity of ideally pure platinum, $\rho_{i,273.15} = 9.82\times10^{-8}\,\Omega\cdot\text{m}$, and then corrected them for thermal expansion. The expansion corrections, i.e., the differences between ρ^* and ρ amount to 0.5% at 800 K, 1% at 1250 K, and 2% at 2000 K.

The recommended values in Table X were obtained by the following procedures. Below 13.8 K, values of $\rho_i(T)$ are in parentheses and represent the behavior of a single platinum resistance thermometer (PTR), normalized by assuming the ice-point value of $9.82 \times 10^{-8}\ \Omega \cdot \text{m}$. Deviations from Matthiessen's rule make it impossible to recommend values in this region.

13.8 to 273.15 K. Recommended values are the mean of data for 34 PTRs, having RRR values ranging from 1500 to 3000. A polynomial function was fitted to values of "reduced" resistance, $Z = (R_T - R_4)/(R_{273.15} - R_4)$. The deviations for individual PTR's decreased from $\pm 2\%$ at 14 K to $\pm 1\%$ at 20 K, 0.1% at 50 K, and 0.01% at 150 K (see also Besley and Kemp [40]).

273.15 to 1300 K. A fifth-order polynomial was fitted to values of W_T (corrected for impurity scattering) for 19 PTRs measured at various fixed points from the ice-point up to the fold point. Uncertainty and scatter in these data taken at four national laboratories reach a maximum of 0.02%.

Above 1300 K. Data were chosen from five sources which satisfied the criterion of fitting smoothly with the PTR data in the range 1200 to 1300 K. As shown in Fig. 19, deviations reach 0.3% above 1800 K. Sources of individual curves are identified in the detailed analysis [39].

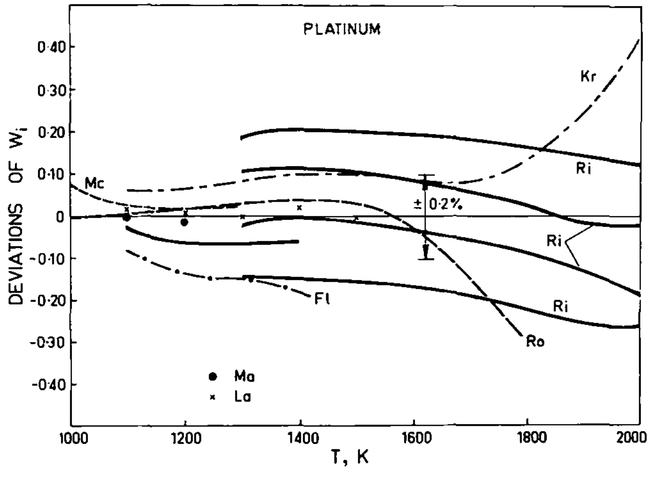

Fig. 19. Platinum. Spread of values of reduced electrical resistance, $W_i = \rho_{iT}/\rho_{i273}$ above 1000 K.

Table X. Values of the Ideal Electrical Resistivity, ρ_i, of Platinum and Tungsten (Corrected for Expansion)[a]

T (K)	ρ_i(Pt) ($10^{-8}\ \Omega\cdot$m)	ρ_i(W) ($10^{-8}\ \Omega\cdot$m)
2	(0.00005_0)	
3	(0.00011_6)	
4	(0.00021_6)	
5	(0.00036)	
6	(0.00057)	
8	(0.00127)	
10	(0.00262)	
12	(0.00501)	
14	0.00915	
16	0.01534	
18	0.02431	
20	0.03669	0.00195
22	0.05296	—
24	0.07352	—
26	0.09864	—
28	0.12848	—
30	0.16310	0.0133
35	0.27007	—
40	0.40381	0.0543
45	0.56006	—
50	0.73411	0.141
55	0.92153	—
60	1.1186	0.266
65	1.3222	—
70	1.5302	0.422
75	1.7409	—
80	1.9531	0.606
85	2.1659	—
90	2.3790	0.809
100	2.8040	1.02
120	3.6462	1.46
140	4.4772	1.89
160	5.2979	2.32
180	6.1107	2.75
200	6.9169	3.18
220	7.7173	3.62
240	8.5125	4.07
260	9.3029	4.52
273.15	9.82	4.82
293.15	10.063	5.28
350	12.805	6.61

[a] $\rho = \rho_0 + \rho_i$.

Table X. *(Continued)*

T (K)	ρ_i(Pt) ($10^{-8}\,\Omega\cdot$m)	ρ_i(W) ($10^{-8}\,\Omega\cdot$m)
400	14.712	7.83
450	16.592	9.08
500	18.445	10.35
550	20.271	11.67
600	22.070	13.00
650	23.843	14.37
700	25.588	15.76
750	27.306	—
800	28.996	18.61
850	30.658	—
900	32.292	21.53
950	33.897	—
1000	35.473	24.51
1050	37.021	—
1100	38.540	27.57
1150	40.03	—
1200	41.50	30.68
1300	44.35	33.84
1400	47.09	37.06
1500	49.74	40.33
1600	52.34	43.65
1700	54.93	47.01
1800	57.51	50.41
1900	60.11	53.85
2000	62.76	57.33
2200		64.41
2400		71.63
2600		79.00
2800		86.51
3000		94.18
3200		102.0
3400		110.0
3600		118.3

Accuracy. For platinum of electrical purity or RRR in the range 1500 to 3000 (corresponding to impurity levels of ~0.01% or less), values of ideal resistivity in Table X (which assume $\rho_{i,273.15} = 9.82 \times 10^{-8}\,\Omega\cdot\mathrm{m}$) should not be in error by more than 2% at 15 K, 1% at 20 K, 0.1% at 50 K, 0.02% at 150–1000 K, 0.1% at 1000–1300 K, and 0.3% at 1300–2000 K. If more accurate determinations of the ice-point resistivity are produced in the future, then all the tabulated values can be multiplied by the appropriate correction factor.

For temperatures above 100 K, a ninth-order polynomial, $10^8 \rho_i = \sum A_n(T/1000)^n \; \Omega \cdot \mathrm{m}$, has been fitted to the tabulated values with deviations of $\leqslant 0.002$ (100–1000 K) and $\leqslant 0.01$ from 1000 to 2000 K. Coefficients are as follows.

$A_0 = -1.621733$

$A_1 = 4.681197 \times 10^1$

$A_2 = -3.258075 \times 10^1$

$A_3 = 8.554023 \times 10^1$

$A_4 = -1.594242 \times 10^2$

$A_5 = 1.837342 \times 10^2$

$A_6 = 1.316886 \times 10^2$

$A_7 = 5.67822 \times 10^1$

$A_8 = -1.340980 \times 10^1$

$A_9 = 1.32990$

4.5. Tungsten

The recommended values for the electrical resistivity of tungsten in Table X were prepared by Chu et al. [38] and are stated to be "for tungsten of purity 99.99% or higher but those below 200 K are applicable only to tungsten having a residual resistivity of $150 \times 10^{-14} \; \Omega \cdot \mathrm{m}$ (RRR ~ 200,000). The estimated uncertainty in the values (of ρ_i) is within ±3% from 100 to 300 K, ±2% between 300 and 2500 K, and ±3% above 2500 K. . . ."

Deviations of selected data from their recommendations are shown in Fig. 20 and suggest that errors are probably less than 2% from 200 to

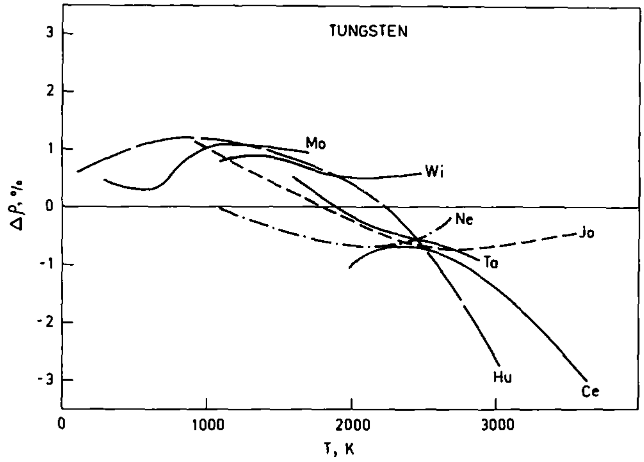

Fig. 20. Tungsten. Percentage deviation of selected electrical resistivity values from recommented values (Table X) [38] above 100 K.

3000 K. These selections are from those listed in earlier analyses by Hust [41, 42] Minges [43], Mundy [44], and Bass [37]. These sources list original references and indicate no significant differences in ideal resistivity among sintered, arc-cast, and zone-refined material at normal and elevated temperatures. At the lowest temperatures ($T < 40$ K), there are large deviations from Matthiessen's rule so that recommendations should be treated with caution.

For convenience of users of data, a fourth-order polynomial has been fitted to the recommended values (corrected for expansion) from 100 to 3600 K with an rms deviation of 0.2% and a maximum deviation of +0.6% at 150 K and −0.5% at 400 K.

The polynomial is $\rho = \sum A_n(T/1000)^n$, where

$$A_0 = -9.68 \times 10^{-1}$$

$$A_1 = 1.9274 \times 10^{1}$$

$$A_2 = 7.826$$

$$A_3 = -1.8517$$

$$A_4 = 2.079 \times 10^{-1}$$

ρ is in $10^{-8}\,\Omega \cdot \mathrm{m}$ and T is in K.

5. THERMAL CONDUCTIVITY

5.1. General Discussion

The thermal conductivity, λ, of a metal or alloy usually is considered to be the sum of the electronic, λ_c, and lattice, λ_g, components:

$$\lambda = \lambda_c + \lambda_g \tag{17}$$

There are other mechanisms of heat transport; however, they generally are not applicable for metals. The electronic term designates the thermal energy carried by the electrons, while the lattice term designates the energy carried by the quantized lattice vibrations (phonons). In pure metals, the lattice term is small (frequently less than 5 and almost always less than 20%). Although theory provides a guide for the dependencies of the lattice conductivity and its order of magnitude, it is generally not sufficient for reference data purposes. Theory does provide an adequate formulation for the low-temperature electronic term. For these reasons, the formulation provided here is based on the theoretical form of the electronic term.

Modifications of this form to account for the lattice contribution and higher temperature dependencies are based on the experimental data. (For some theoretical background, see, for example, Berman [45] and Klemens [46]).

Theory shows that at low temperatures the electronic thermal conductivity component is limited mainly by two mechanisms: (a) the interaction of the electrons with phonons and (b) the interaction of the electrons with physical and chemical imperfections. The interaction of electrons with phonons leads to a resistive term approximately proportional to T^2. The proportionality constant, α, is characteristic of the generic type of metal, i.e., it is an intrinsic property of the base metal. The interaction of electrons with lattice defects leads to a thermal resistivity which is inversely proportional to temperature. The proportionality constant, β, is determined by the type and concentration of the lattice imperfections. The approximation of low-temperature electronic thermal conductivity is written as

$$\lambda = (W_0 + W_i)^{-1} = (\beta/T + \alpha T^n)^{-1} \tag{18}$$

where $n = 2$ and where W_0 represents the electron-defect interaction and W_i is the electron–phonon interaction. Cezairliyan and Touloukain [47] presented a revised from of this theoretical equation to account for observed deviations. This revised from, reviewed in Vol. 1 of Ref. 17, is based on experimental data that show that (a) n is usually larger than 2, and (b) α is weakly dependent on lattice imperfections. The modified equation is valid only for temperatures up to about 1.5 times the temperature at which the maximum in thermal conductivity occurs. For the metals of interest here, this limit occurs at about 40 K. At higher temperatures, thermal conductivity decreases more slowly with increasing temperature than predicted by this equation. Theory predicts that at high temperatures, the thermal conductivity of metals should approach a constant. To account for this high-temperature behavior, the form presented by Cezairliyan and Toulouklan [47] has been modified. Finally, evidence has been presented that indicates the presence of an interaction term between W_0 and W_i. It is denoted W_{i0}. The base equation selected to represent the predominant thermal conductivity behavior of these metals is, therefore,

$$\lambda_b = (W_0 + W_i + W_{i0})^{-1} \tag{19}$$

where

$$W_0 = \beta/T \tag{20}$$

$$W_i = P_1 T^{P_2}/(1 + P_1 P_3 T^{(P_2 + P_4)} \exp(-P_5/T)^{P_6})) + W_c \tag{21}$$

$$W_{i0} = P_7 W_i W_0 (W_i + W_0) \tag{22}$$

where the P_i's are parameters determined by least-squares fit of the experimental data. The quantity W_c is a temperature-dependent term (defined later for each metal) that accounts for mathematical residual deviations in W_i. This equation is intended to describe the thermal conductivity of annealed bulk specimens of these metals. Thus, it describes the intrinsic thermal conductivity and the limiting effct of the presence of chemical impurities in each metal. The reader should be a aware, however, of the existence of other limiting mechanisms, such as physical defects, size effects, and magnetic field effects.

Both the thermal conductivity and the electrical resistivity of a pure metal are strongly dependent on the concentration of lattice imperfections. Indeed, they are both influenced to the same degree by the imperfections at very low temperatures. The resulting correlation is referred to as the Widemann–Franz–Lorenz law. Since electrical resistivity is much easier to measure accurately than thermal conductivity, it is often used to determine imperfection concentrations and, therefore, thermal conductivity.

5.2. Literature Review

The existing principal compilations and reviews [17, 48] were used as a starting point for this compilation. The resulting list of references was updated by searching current literature, abstracting services, and computerized data banks, as well as the reference lists of the most recent publications. The initial emphasis was directed toward temperatures below 300 K. Later, the scope was extended to include temperatures up to near the melting point of each metal. The high-temperature compilation was directed toward obtaining the most significant publications rather than a complete listing.

Since our principal interest is the dependence of thermal conductivity on temperature and electrical resistivity for relatively pure metals, not all of the literature data for a given base metal are referenced in Ref. 49. For example, numerous publications on the measurement of thermal conductivity at a single temperature have been excluded. Also, all measurements on specimens with more than 1% total impurity concentration were excluded.

Each of the selected sources was coded for content, and the data were extracted for computer analysis. When the literature data were presented in graphical form, the graphs were enlarged and read as accurately as possible. The resulting data were then smoothed to reproduce the original curves. Each set of data for a given measured specimen was characterized with values of residual resistivity, RRR, chemical impurity concentrations, and thermal/mechanical history. Other details regarding the experimental procedure, purpose of the work, and analysis of data are also coded in the annotated bibliography in Ref. 49 for the convenience of the reader.

5.3. Analysis

Selected data sets (primary data) were used to establish best values over the entire range of temperature and ρ_0 or RRR. The primary data were chosen on the basis of proven laboratory techniques, as applied to pure and annealed specimens. These data were believed to be accurate to within 5 to 10%. The primary data were then used to optimize the parameters in the selected equation for each metal. The ρ_0 or RRR value assigned to each data set was not necessarily that reported by the author. The values were selected to minimize the thermal conductivity deviations in the low-temperature range, i.e., the range below the peak in the curve. If data did not exist below the peak in the λ vs T curve, the author's value of RRR (or ρ_0) was used. If a value of RRR (or ρ_0) was not reported for a high-temperature data set, it was estimated by considering purity and anneal conditions. The relationship of the selected values of RRR (or ρ_0) and reported values are shown in Ref. 49. The fitted equation was then compared to other data sets, including those for the less pure and unannealed specimens. The comparisons were examined for deviations varying systematically with RRR and temperature. The results of this analysis are given in Ref. 49.

The relationship between RRR and ρ_0 is not uniquely defined for a given specimen in the absence of ice-point resistivity measurements. For very pure metals, RRR is frequently reported because the determination of RRR can be done without a knowledge of the form factor, $1/A$, i.e., the length-to-cross-sectional area ratio of the specimen. Such specimens are frequently very thin; consequently, the accurate determination of $1/A$ and ρ_0 is difficult. In the absence of Matthiessen's rule (MR) deviations, RRR $= (\rho_{i273}/\rho_0) + 1$ may be used to defined the relationsip between ρ_0 and RRR, where ρ_{i273} is the ice-point intrinsic resistivity. However, MR deviations are known to exist, and thus this equation is also inexact. We have chosen the following procedure to establish values of ρ_0 and RRR for each thermal conductivity measurement in the literature:

a. The value of ρ_0 that produces the best agreement of Eq. (19) and the reported thermal conductivity data at low temperatures was selected. The corresponding residual resistivity ratio, RRR, was calculated from the above equation. In addition, at this stage of the analysis, the Sommerfeld value of the Lorentz ratio, $L_0 = 2.443 \times 10^8$, was assumed to be valid to obtain the corresponding value of $\beta = \rho_0/L_0$.

The values of ρ_{i273} (in $\mathrm{n}\Omega \cdot \mathrm{m}$) used for the four metals investigated are as follows: 15.4 for Cu, 24.8 for Al, 87.0 for Fe, and 48.4 for W.

The above values listed are best estimates based on literature values and a variety of published and unpublished measurements performed at the National Institute of Standards and Technology over a period of 25 years.

> b. Upon completion of the data fits and comparisons, plots were made of RRR (selected) versus RRR (reported) for each metal. These plots were used to determine the Lorenz ratio adjustment necessary for each metal and the range of RRR for which Eq. (19) is valid. Recommended values as a funtion of T and RRR are given in the form of equations, tables, and graphs for each metal.

5.4. Thermal Expansion Corrections

It would be desirable to specify that the recommended values in this report are based on the actual dimensions of the specimens at any given temperature, i.e., they are corrected for thermal expansion. However, only a small fraction of the authors indicated whether their reported data were corrected or uncorrected for thermal expansion. Therefore, the reported data were not modified for thermal expansion effects. Although an undesirable situation, this is not serious in view of the relatively large measurement uncertainties. It is also noted that, although a linear correction is usually appropriate, the proper correction depends on the nature of the measurement method employed.

Room temperature is the basis for this correction since specimen dimensions are normally determined at room temperature. The four metals discussed have positive thermal expansion coefficients, thus the thermal conductivity values corrected for thermal expansion are smaller than the uncorrected values at temperatures above room temperature and larger at temperatures below room temperature. A more detailed presentation can be found in Ref. 49.

5.5. Aluminium

Aluminium has been measured extensively but not as completely as copper. A total of 35 publications is included in the annotated bibliography in Ref. 49. Of these, nine data were selected as primary data.

The primery data are for annealed specimens only and cover a range of temperature from 2 to 873 K and a range of RRR from 13 to 16,800. Thermal conductivity values for these sources are shown in Fig. 21. Although the RRR of the most electrically pure aluminium ever produced is comparable to that for copper (50,000), aluminium, with RRR values in the range of 10,000 to 20,000, is more readily obtained.

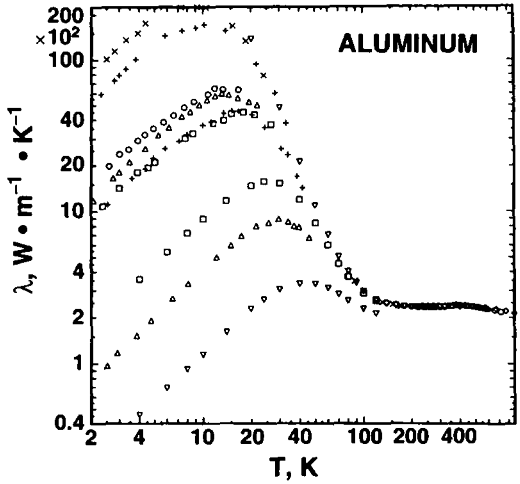

Fig. 21. Aluminum. Primary thermal conductivity data (see Ref. 49 for identification of sources).

Equation (19) was fitted to the primary data set over the entire temperature range. The values of the parameters, P_i, $i = 1, 2, \ldots, 7$, obtained by nonlinear least-squares fit are as follows:

$P_1 = 4.716 \times 10^{-8}$

$P_2 = 2.446$

$P_3 = 623.6$

$P_4 = -0.16$

$P_5 = 130.9$

$P_6 = 2.5$

$P_7 = 0.8169$

with all units in SI.

The data at high RRR were then examined for systematic residual deviations as a function of temperature. These residuals were represented by the W_c term in Eq. (21). The resulting equation for W_c is

$$W_c = -0.0005 \ln(T/330) \exp(-(\ln T/380)/0.6)^2) - 0.0013 \ln(T/110) \exp(-(\ln T/94)/0.5)^2) \qquad (23)$$

where W_c is in mK · W^{-1} and T is in K.

The overall primary aluminium data yields random deviations from Eq. (19). Some of the individual data sets exhibit systematic trends. The deviations of these data are shown in Fig. 22. Although temperature-dependent deviations exist for individual data sets, the overall pattern is random in nature. No systematic trends with RRR were noted.

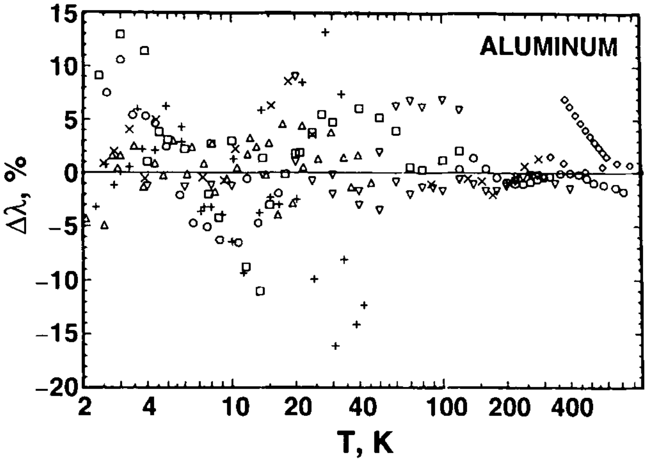

Fig. 22. Aluminum. Deviations of primary thermal conductivity data from Eq. (19).

The primary data were selected from the literature data on relatively large, well-annealed specimens. Therefore, the deviations exhibited in Fig. 22 are indicative of the combined effect of (a) experimental measurement errors and (b) the inability of Eq. (19) to account for the effects of chemical impurity variations. The effects of physical defect variations, small size variations, and magnetic fields are exhibited, in part, by the deviations of the secondary data. The thermal conductivity variations caused by other than chemical impurity variations are not expected to be represented as well by Eq. (19). However, the RRR (or ρ_0) correlating parameter does account for an appreciable part of these variations as well. Equation (19) is compared to the most common sources of reference data [50]. The agreement is within the combined uncertainties except in the region around 60 K for which data were published after the compilations [21, 50]. Thermal conductivity values calculated from Eq. (19) are shown in Fig. 23 and Table XI.

5.6. Copper

Copper is the most extensively measured metal. A total of 44 references was selected for inclusion in the annotated bibliography in Ref. 49. Twenty-two of these references represent the primary data sets.

The primary data cover a range of temperatures from 0.2 to 1250 K and a range of RRR from 20 to 1800. These data are illustrated in Fig. 24.

Table XI. Thermal Conductivity Values for Aluminum Calculated from Eq. (19) at Selected Temperatures and RRR Values

	λ (W · m^{-1} · K^{-1})					
T(K)	RRR = 30	RRR = 100	RRR = 300	RRR = 1,000	RRR = 3,000	RRR = 10,000
1	29	98	295	984	2,954	9,842
2	57	195	589	1,966	5,892	19,421
3	86	292	883	2,941	8,765	28,499
4	114	390	1,175	3,897	11,475	35,887
5	143	487	1,463	4,817	13,885	40,840
6	171	583	1,746	5,677	15,853	43,072
7	200	678	2,020	6,452	17,272	42,980
8	228	772	2,282	7,116	18,109	41,300
9	256	864	2,528	7,651	18,406	38,717
10	284	953	2,755	8,044	18,260	35,708
12	338	1,122	3,133	8,420	17,095	29,474
14	391	1,272	3,398	8,340	15,360	23,801
16	442	1,400	3,544	7,960	13,478	19,074
18	489	1,500	3,582	7,418	11,658	15,308
20	532	1,572	3,534	6,801	9,997	12,366
25	617	1,628	3,178	5,227	6,706	7,527
30	662	1,542	2,666	3,843	4,492	4,791
35	664	1,373	2,130	2,756	3,036	3,151
40	631	1,172	1,652	1,972	2,096	2,143
45	581	980	1,274	1,440	1,497	1,519
50	526	817	997	1,087	1,117	1,128
60	430	588	664	696	706	710
70	361	454	492	507	512	513
80	312	372	394	403	405	406
90	278	320	334	340	341	342
100	255	286	297	300	301	302
150	223	239	244	245	246	246
200	222	234	237	238	239	239
250	224	233	235	236	237	237
300	226	234	236	237	237	237
400	231	237	239	239	239	239
500	230	235	237	237	237	237
600	226	230	231	231	231	231
700	220	229	224	224	224	224
800	214	217	217	218	218	218
900	209	212	212	212	212	212

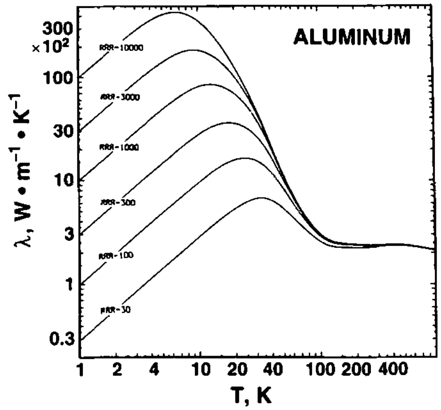

Fig. 23. Aluminum. Thermal conductivity as a function of temperature calculated from Eq. (19) at selected values of RRR.

The RRR of typical commercially pure copper wire is in the range of 50 to 500. A very high-purity copper, produced routinely, may have an RRR as high as 2000.

Equation (19) was fitted to represent the primary copper data over the entire temperature range.

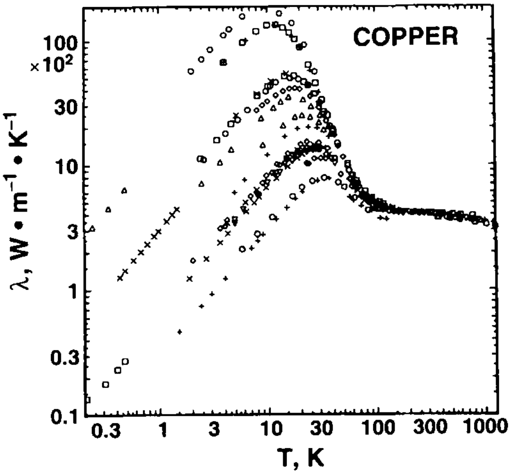

Fig. 24. Copper. Primary thermal conductivity data (see Ref. 49 for data sources).

The values of the parameters, P_i, $i = 1, 2, \ldots, 7$ [Eq. (21)], obtained by nonlinear least-squares fit are as follows:

$$P_1 = 1.754 \times 10^{-8} \qquad P_5 = 70$$

$$P_2 = 2.763 \qquad P_6 = 1.765$$

$$P_3 = 1102 \qquad P_7 = 0838/\beta_r^{0.1661}$$

$$P_4 = -0.165$$

where $\beta_r = \beta/0.0003$. All units are in SI.

The data at high RRR were examined for systematic residuals as a function of temperature. These residuals were represented by the W_c term in Eq. (21). The result is

$$\begin{aligned} W_c = & -0.00012 \ln(T/420) \exp(-(\ln T/470/0.7)^2) \\ & -0.00016 \ln(T/73) \exp(-(T/87)/0.45)^2) \\ & -0.00002 \ln(T/18) \exp(-(T/21)/0.5)^2) \end{aligned} \tag{24}$$

where W_c and T are in SI units.

The deviations of the primary data from Eq. (19) are illustrated in Fig. 25. Five data sets exhibit differences of greater than ±10%. In most of these cases, the deviations are significantly higher than the stated or implied uncertainty of the source document. It is not clear if these deviations

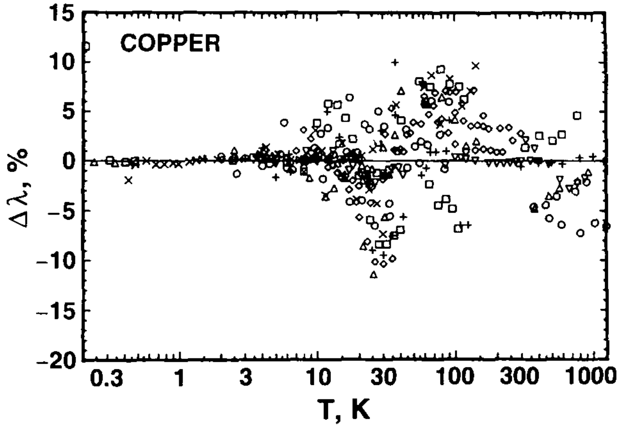

Fig. 25. Copper. Deviations of primary thermal conductivity data from Eq. (19).

are the result of understimated uncertainties or the result of real differences between specimens. Although temperature-dependent deviations do exist for individual data sets, the overall pattern is random in nature. No systematic trends with RRR were identified.

It was of interest to compare the values calculated from Eq. (19) to the most widely used references [17, 49] of recommended thermal conductivity values. Within the combined uncertainties, this work and Refs. 17 and 49 give values only for a single value of RRR (approximately 1800), while Eq. (20) covers a wide range of RRR.

Thermal conductivity values calculated from Eq. (19) are illustrated in Fig. 26 and listed in Table XII.

5.7. Iron

A total of 41 references on iron is included in the annotated bibliography in Ref. 49. Of these, 15 were chosen as primary data sets.

The primary data cover a range of temperatures from 1.5 to 1000 K and a range of RRR from 4 to 200. These data are illustrated in Fig. 27. Iron produced in bulk form is generally of much lower electrical purity than either copper or aluminium. RRR values above 550 are not reported.

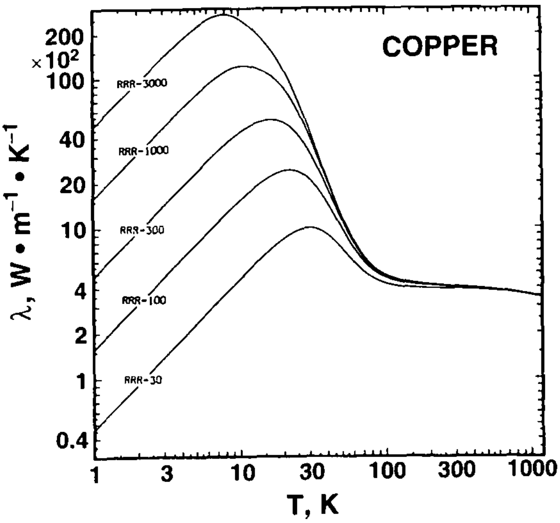

Fig. 26. Copper. Thermal conductivity as a function of temperature calculated fromc Eq. (19) at selected values of RRR.

Table XII. Thermal Conductivity Values for Copper Calculated from Eq. (19) at Selected Temperature and RRR Values

	λ (W · m^{-1} · K^{-1})				
T (K)	RRR = 30	RRR = 100	RRR = 300	RRR = 1,000	RRR = 3,000
1	46	156	471	1,574	4,726
2	91	312	942	3,147	9,434
3	137	468	1,413	4,710	14,044
4	183	624	1,880	6,243	18,380
5	228	779	2,343	7,715	22,170
6	274	933	2,796	9,075	25,084
7	319	1,085	3,232	10,260	26,834
8	365	1,235	3,642	11,197	27,328
9	409	1,390	4,015	11,836	26,756
10	454	1,520	4,343	12,172	25,496
12	541	1,778	4,844	12,127	22,264
14	624	2,002	5,144	11,544	19,150
16	703	2,186	5,267	10,725	16,398
18	777	2,324	5,231	9,771	13,924
20	843	2,408	5,054	8,727	11,683
25	960	2,381	4,215	6,135	7,271
30	999	2,119	3,245	4,151	4,573
35	970	1,784	2,436	2,859	3,028
40	900	1,467	1,841	2,047	2,122
45	814	1,205	1,423	1,531	1,568
50	731	1,002	1,135	1,196	1,216
60	597	740	799	824	832
70	513	601	634	647	651
80	465	526	549	557	560
90	437	485	502	508	510
100	421	461	475	480	482
150	396	419	426	429	430
200	391	407	413	414	415
250	388	401	405	407	407
300	386	397	400	401	402
400	383	391	393	394	394
500	379	385	387	388	388
600	374	379	381	381	381
700	368	372	374	374	374
800	362	365	367	367	367
900	356	359	360	360	360
1,000	350	352	353	353	354
1,100	344	347	347	348	348
1,200	339	341	342	342	342
1,300	335	337	337	338	338

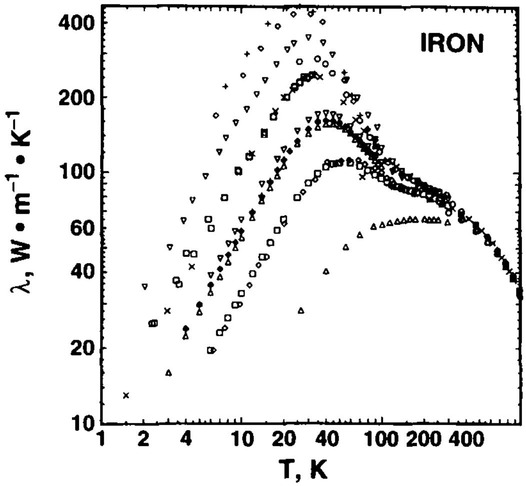

Fig. 27. Iron. Primary thermal conductivity data (see Ref. 49 for data sources).

Equation (19) was used to represent the iron data over the entire temperature range. The values for the parameters, P_i, $i = 1, 2, \ldots 7$, obtained by nonlinear least-squares fit are as follows:

$$P_1 = 166.9 \times 10^{-8} \qquad P_5 = 238.6$$

$$P_2 = 1.868 \qquad P_6 = 1.392$$

$$P_3 = 1.503 \times 10^5 \qquad P_7 = 0.0$$

$$P_4 = -1.22$$

where all units are in SI.

The data at high RRR were examined for systematic residual deviations as a function of temperature. The residuals were represented by the W_c term in Eq. (21) with the following equation:

$$W_c = -0.004 \ln(T/440) \exp(-(\ln T/650/0.8)^2) - 0.002 \ln(T/90) \exp(-(T/90)/0.45)^2) \qquad (25)$$

where W_c and T are in SI units.

The deviations of the primary data from Eq. (19) are illustrated in Fig. 28. Only three data sets exhibit differences greater than $\pm 10\%$. Although deviations, systematic with temperature, exist for individual data

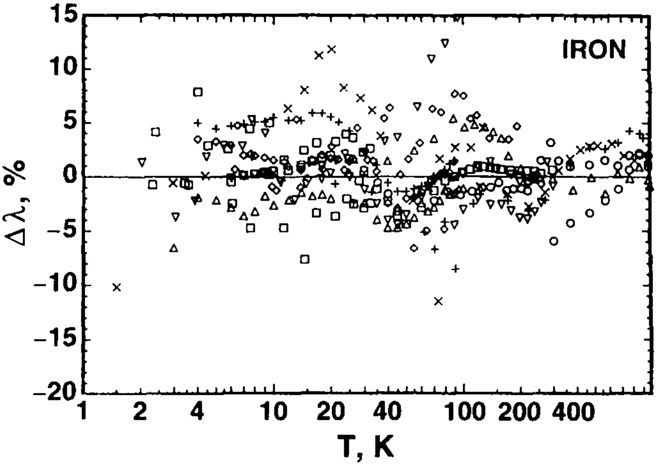

Fig. 28. Iron. Deviations of primary thermal conductivity data from Eq. (19).

sets, the overall pattern is random in nature. No systematic trends varying with RRR were identified.

The equation developed here is compared to the reference data in Refs. 21 and 50–52. The differences are within the combined uncertainties of the sources. It is noted that data within the primary set for RRR above

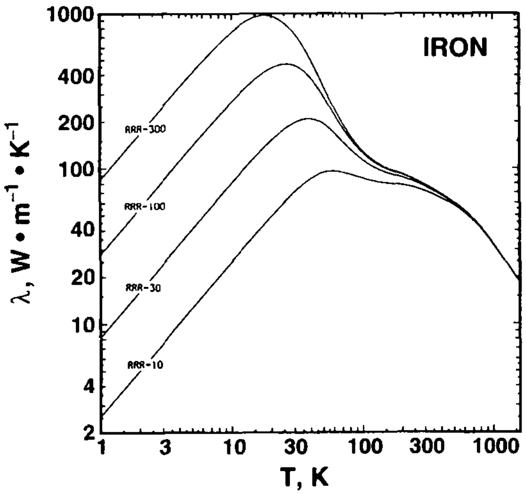

Fig. 29. Iron. Thermal conductivity as a function of temperature calculated from Eq. (19) at selected values of RRR.

Table XIII. Thermal Conductivity Values for Iron Calculated from Eq. (19) at Selected Temperatures and RRR Values

	$\lambda\,(\mathrm{W\cdot m^{-1}\cdot K^{-1}})$			
T (K)	RRR = 10	RRR = 30	RRR = 100	RRR = 300
1	2.5	8.1	28	84
2	5.1	16.3	56	168
3	7.6	24	83	251
4	10.1	32	111	333
5	12.6	41	138	414
6	15.2	49	166	492
7	17.7	57	192	567
8	20	65	218	637
9	23	73	244	702
10	25	81	269	761
12	30	96	315	858
14	35	111	357	925
16	40	125	393	961
18	45	139	422	970
20	49	152	445	957
25	61	179	471	863
30	71	198	462	735
35	79	208	429	609
40	86	210	384	500
45	91	204	336	410
50	94	194	292	340
60	96	170	225	247
70	94	150	183	195
80	92	133	156	164
90	89	122	138	144
100	87	114	126	130
150	81	94	100	102
200	78	88	91	92
250	76	82	85	85
300	72	77	79	79
400	64	67	68	69
500	58	60	60	61
600	52	53	54	54
700	46	47	47	47
800	41	41	42	42
900	36	37	37	37
1000	32	32	33	33

200 were not included. Therefore the comparison to the data in Refs. 17 and 50 is an extrapolation of the range of validity of Eq. (19). Thermal conductivity values calculated from Eq. (19) are illustrated in Fig. 29 and listed in Table XIII.

5.8. Tungsten

The annotated bibliography for tungsten in Ref. 49 includes 39 references. Of these data sets, 13 were selected as primary data.

The primary data range in temperature from 2 to 3000 K and in RRR from 30 to 170. The primary data are shown in Fig. 30.

Because high-purity, single-crystal tungsten specimens exhibit unusual behavior at the lowest temperatures, these data are not included in the primary data set. As a consequence, the range of RRR included in the fit of the data is more restricted than the total range of data.

Equation (19) was fitted to the primary data oer the entire range of temperatures. The values of the parameters, P_i, $i = 1, 2, \ldots 7$, obtained by nonlinear least-squares fit are as follows:

$$\begin{array}{ll} P_1 = 31.70 \times 10^{-8} & P_5 = 69.94 \\ P_2 = 2.29 & P_6 = 3.557 \\ P_3 = 541.3 & P_7 = 0.0 \\ P_4 = -0.22 & \end{array}$$

where all units are in SI.

The systematic residuals for this equation were represented by the W_c term in Eq. (21). The result is

$$\begin{aligned} W_c = {} & -0.00085 \ln(T/130) \exp(-(\ln T/230/0.7)^2) \\ & -0.00015 \exp(-(T/3500)/0.8)^2) \\ & -0.0006 \ln(T/90) \exp(-(\ln(T/80)/0.4)^2) \\ & -0.00003 \ln(T/24) \exp(-(\ln(T/33)/0.5)^2) \end{aligned} \tag{26}$$

where W_c and T are in SI units.

The deviations of the primary data from Eq. (19) with these parameters are illustrated in Fig. 31. No deviations exhibit differences greater than ±7%. Although there are systematic trends with respect to temperature, the overall pattern is random in nature. No systematic trends varying RRR were identified.

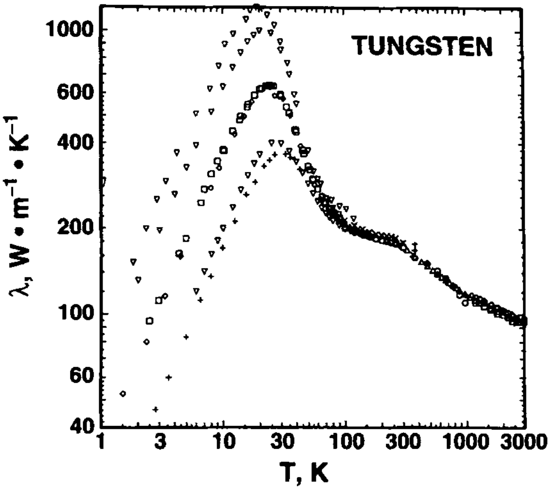

Fig. 30. Tungsten. Primary thermal conductivity data (see Ref. 49 for data sources).

Again, it was of interest to compare this equation to existing reference data. The data from Ref. 50 for Standard Reference Material (designated SRM 730) at values of RRR = 50, 75, and 100 agree to within 5%. The reference data from Refs. 42 and 50 are for an RRR of 2850 and therefore represent an extrapolation of the equation as given here. The differences between the recommended values of Refs. 50 and 42 are about 30%

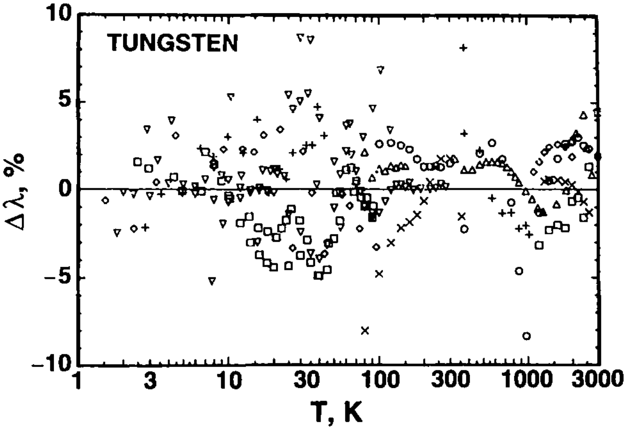

Fig. 31. Tungsten. Deviations of primary thermal conductivity data from Eq. (19).

Table XIV. Thermal Conductivity Values for Tungsten Calculated from Eq. (19) at Selected Temperatures and RRR Values

	λ $(W \cdot m^{-1} \cdot K^{-1})$		
T (K)	RRR = 30	RRR = 100	RRR = 300
1	14.6	50	151
2	29	100	302
3	44	150	452
4	59	200	602
5	73	249	749
6	88	299	894
7	102	347	1033
8	117	395	1166
9	131	442	1291
10	145	488	1404
12	173	574	1595
14	201	651	1730
16	227	718	1802
18	251	768	1803
20	273	799	1734
25	311	786	1378
30	325	692	1020
35	321	586	768
40	306	494	600
45	285	418	483
50	262	357	398
60	226	281	302
70	211	250	264
80	204	236	246
90	199	225	234
100	195	217	224
150	184	197	201
200	180	189	191
250	175	182	184
300	169	174	176
400	155	158	159
500	143	145	146
600	135	136	137
700	129	130	130
800	124	125	126
900	121	122	122
1000	118	119	119
1100	115	116	116
1200	113	114	114
1300	111	111	112
1400	109	110	110
1500	107	108	108
1600	106	106	106
1800	103	103	103
2000	100	101	101
2200	98	99	99
2400	96	97	97
2600	95	95	95
2800	93	93	93
3000	92	92	92

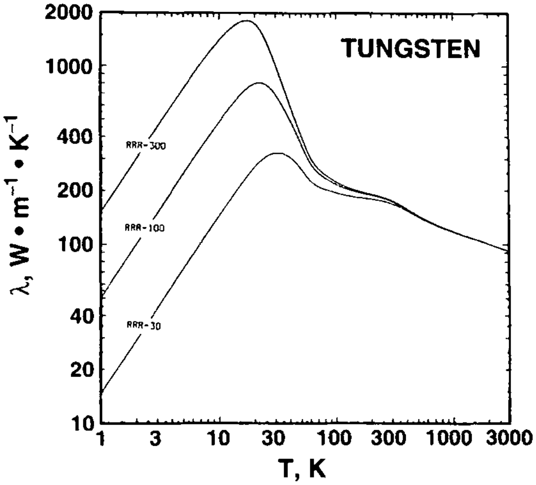

Fig. 32. Tungsten. Thermal conductivity as a function of temperature calculated from Eq. (19) at selected values of RRR.

between 10 and 20 K. The differences otherwise are within the combined uncertainties of the reference data.

Thermal conductivity values calculated from Eq. (19) are illustrated in Fig. 32 and listed in Table XIV.

6. ABSOLUTE THERMOPOWER

To determine the absolute thermoelectric power, S, of a material, it is necessary to measure its Thomson coefficient, μ, from absolute zero to the temperature of interest as

$$S(T) = \int_0^T (\mu/T)\, dT \tag{27}$$

The Thomson heat is the heat absorbed (or evolved) when electrons move through a temperature gradient in the material. It is sometimes called "the specific heat of electrons" and is characteristic of the material. One way to measure it is to observe the change in temperature profile of the material when the direction of the current is reversed. Because the Thomson heat is small compared with the Joule (resistive) heat, the measurements are difficult and scarce.

When the thermopower of a reference mterial (A) has been measured, it is then relatively simple to measure the (differential) thermopower of a pair of materials, S_{AB}, and hence find the thermopower of material B as

$$S_B = S_{AB} + S_A \tag{28}$$

At sufficiently low temperatures a superconductor can be chosen for the reference material (as $S_A = 0$ in the superconducting state). Above the superconducting range and below room temperature, the material chosen is lead. The thermopowers of other materials are more dependent on their metallurgical history (impurity levels, strain, grain size, crystal orientation, etc.). For example, the thermopowers of the noble metals (Cu, Ag, and Au) are extremely sensitive to magnetic impurities at low temperatures.

The thermopower scale for lead by Christian et al. [53] was based on their own measurements of $S_{\mathrm{Pb/superc.}}$ and a series of measurements of S_{Pb} done by Borelius and his group from 20 to 290 K in the period 1910 to 1932 [54]. Nystrom [55] measured μ_{Cu} directly from 650 to 1070 K and used the earlier Borelius data to generate scales of S_{Cu} and S_{Pt} up to 1300 K. Rudnitskii [56] assembled the data of Nystrom and Borelius in a different way, from which he predicted S_{Pt} at high temperatures. Lander [57] made separate measurements of the Thomson heat μ for Ag, Au, Cu,

Table XV. Measurements of Thomson Heat, μ, and Thermopower, S

Temperature range (K)	Measurements	Tables	Ref. No.
20–290	μ(AgAu); V(AgAu/Pb) V(AgAu/Sn)	μ(AgAu, Sn, Pb) S(AgAu, Sn, Pb)	54
7–18	V(Pb/Nb_3Sn)	S(Pb)	53
650–1070	μ(Cu, PtCu) V(Cu/Pt)	μ(Cu) S(Cu, Pt)	55
100–m.p.		S(Cu, Ag, Au, Pt, Pd, W, Mo)	58
4–350	μ(Pb, AgAu) V(Pb/AgAu)	S(Pb)	64
300–550 300–850	μ(Pb, Cu); V(Pb/Cu) μ(Cu); V(Cu/Pt); V(Cu/Au)	S(Pb) S(Cu, Pt, Au)	65
400–m.p.	μ(Cu, Ag, Au, Pt, Pd, W, Mo)	μ(Cu, Ag, Au, Pt, Pd, W, Mo)	57
900–1700	μ(Pt, W)	S(Pt, W)	7

Table XVI. Absolute Thermopower for Lead, Copper, Platinum, and Tungsten (s, Superconducting; n, Normal)

T (K)	S(Pb) (μV · K^{-1})	T (K)	S(Pb) (μV · K^{-1})
0.0	0.000	30	−0.657
7.2(s)	0.000	32	−0.636
7.2(n)	−0.200	34	−0.617
7.5	−0.220	36	−0.601
8.0	−0.256	38	−0.587
9.0	−0.343	40	−0.575
9.5	−0.388	45	−0.551
10.0	−0.433	50	−0.537
10.5	−0.476	55	−0.530
11.0	−0.517	60	−0.527
11.5	−0.556	70	−0.531
12.0	−0.593	80	−0.544
13.0	−0.657	90	−0.562
14.0	−0.707	100	−0.583
15.0	−0.745	120	−0.631
16.0	−0.770	140	−0.682
17.0	−0.782	160	−0.734
18.0	−0.786	180	−0.785
20.0	−0.779	200	−0.834
22.0	−0.760	220	−0.882
24.0	−0.735	240	−0.927
26.0	−0.707	260	−0.969
28.0	−0.681		

T (K)	S(Pb) (μV · K^{-1})	S(Cu) (μV · K^{-1})	S(Pt) (μV · K^{-1})	S(W) (μV · K^{-1})
273	−1.00	1.79	−4.04	0.77
300	−1.05	1.94	−4.92	1.44
350	−1.16	2.22	−6.33	2.95
400	−1.28	2.50	−7.53	4.62
450	−1.41	2.78	−8.59	6.26
500	−1.56	3.07	−9.53	7.85
550	−1.73	3.35	−10.41	9.34
600		3.62	−11.22	10.75
650		3.89	−11.98	12.08
700		4.16	−12.71	13.33
750		4.43	−13.42	14.48
800		4.70	−14.14	15.51
850		4.96	−14.89	16.40
900		5.23	−15.66	17.19
1000			−17.21	18.46
1100			−18.77	19.40
1200			−20.29	20.06
1300			−21.78	20.45
1400			−23.18	20.63
1500			−24.49	20.70
1600			−25.67	20.61
1700				20.12
1800				19.15

Mo, Pd, Pt, and W from about 400 K to their respective melting points, using an indirect method. Cusack and Kendall [58] combined the Lander data with those of Borelius to generate tables of S for various metals. Inconsistencies in these scales led others [59–63] to generate their own scales for S_{Pt} and/or S_{Pb} on the basis of various relative thermopower measurements. All of these scales, however, were derived ultimately from the Borelius measurements.

In 1977, Roberts [64] made direct measurements of μ_{Pb} and found that there was a small error in the Borelius values between 20 and 60 K, having the effect that the whole scale for S_{Pb} was too low by about $0.3\,\mu V \cdot K^{-1}$ above about 50 K. Roberts [65] extended his measurement of μ fo Pb and for Cu and obtained thermopower scales which were self-consistent within $0.05\,\mu V \cdot K^{-1}$ to 600 K. His values for μ_{Cu} agreed well with those of Nystrom [55].

Later, Roberts et al. [7] measured μ_{Pt} and μ_W from 900 to 1700 K. Their values agreed generally with those of Lander but showed less scatter.

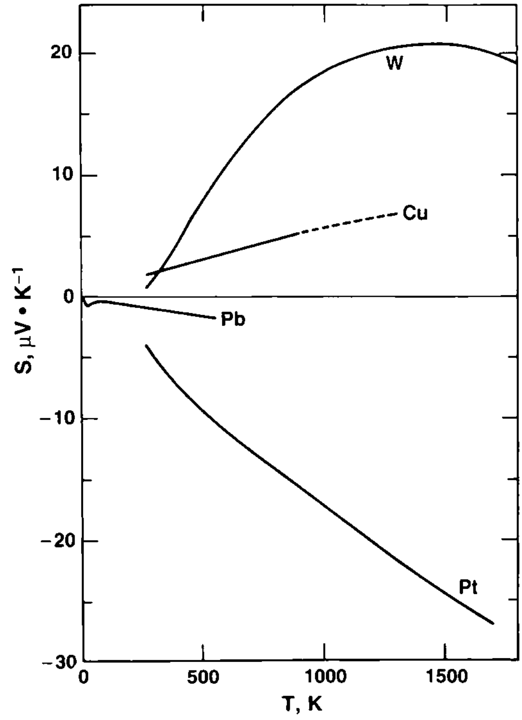

Fig. 33. Absolute thermopower of reference metals: lead, copper, platinum, and tungsten.

In terms of thermopower, the differences in $\int (\mu/T)\,dT$ amounted to a few tenths of a $\mu V \cdot K^{-1}$ for Pt.

Roberts et al. [7] were able to cross check their measurements of μ on Pt and W by also measuring the thermopower $S_{Pt/W}$ and showed that errors in S_{Pt} at 1500 K were less than $0.2\,\mu V \cdot K^{-1}$. Their scale for S_{Pt} is in remarkably good agreement with the predictions of Rudniiskii [46] and about $0.6\,\mu V \cdot K^{-1}$ above the Cusack and Kendall [58] values.

Table XV summarizes the principal measurements of μ and S, while Table XVI and Fig. 33 give values of the absolute thermopowers of Pb below the ice point and Pb, Cu, Pt, and W above the ice point, based on the work of Roberts and his collaborators [7, 64, 65].

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